Setting a Robust SLAM problem in a set of equations

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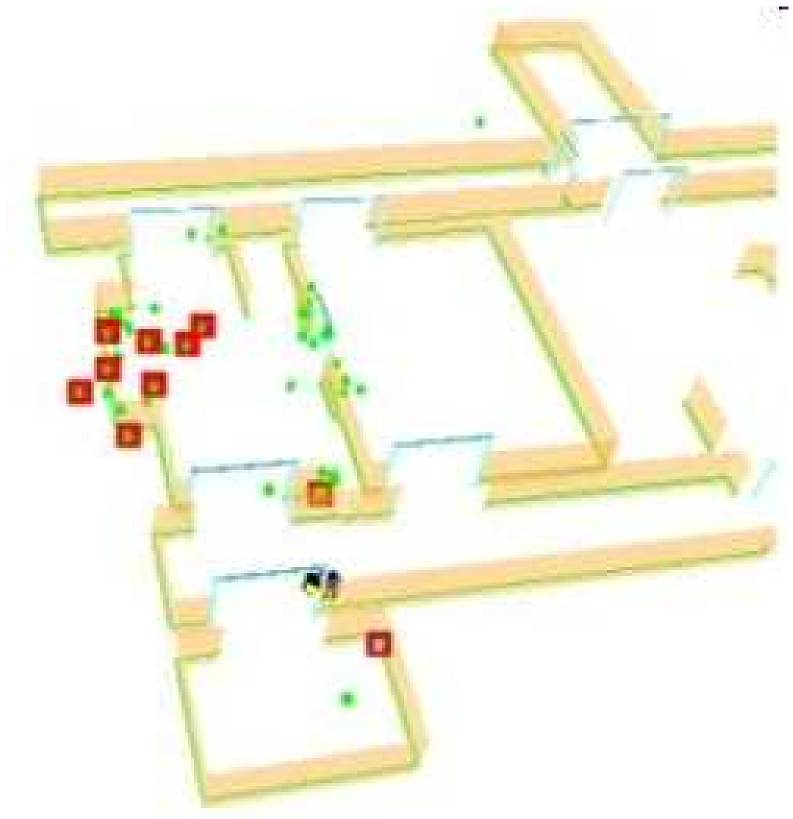
SLAM

- SLAM: Simultaneous Localization And Mapping
- **non-linear** equations
- **Outliers** in the data
- Areas determined by **polygons** and **segments**



Example of a robots movement area

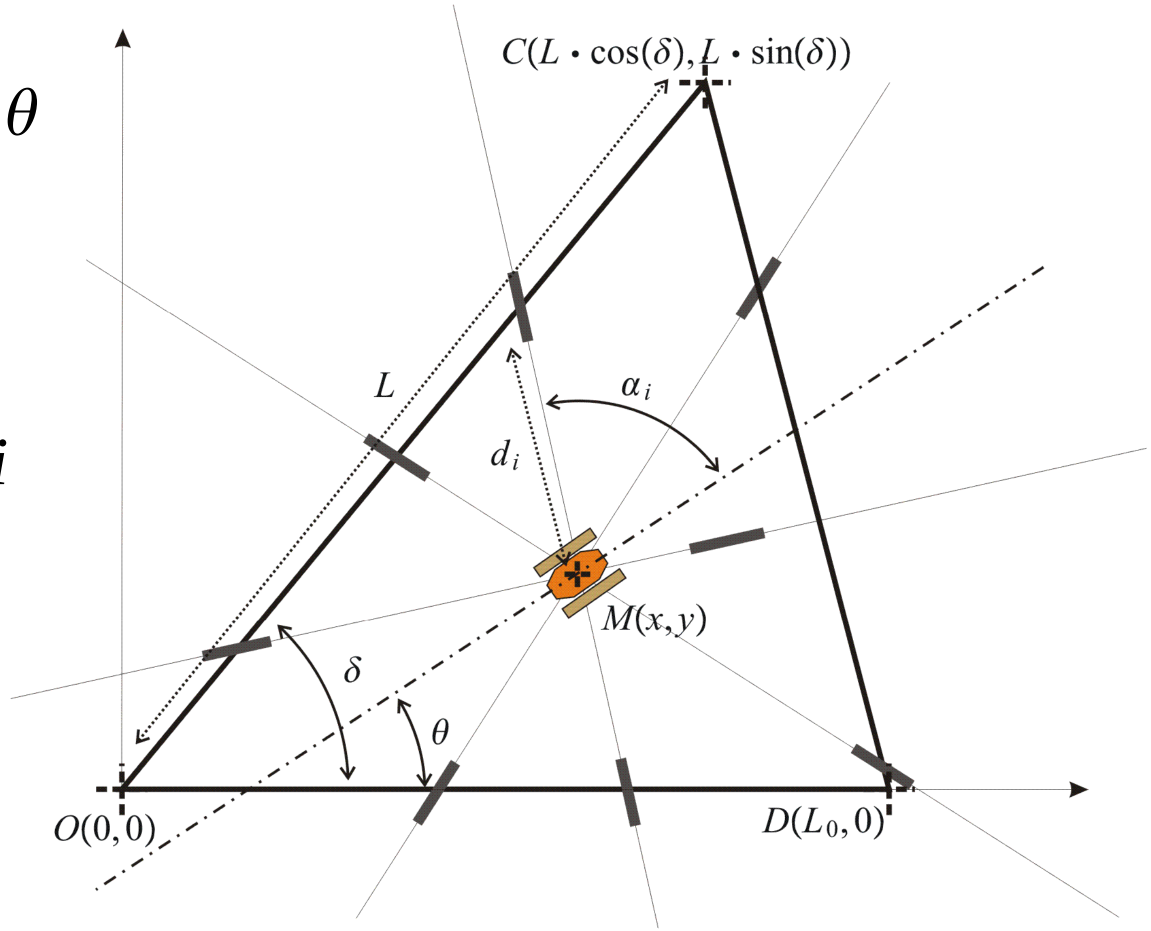
Simplified Problem

Unknown factors

- position x, y, θ
- map L, δ

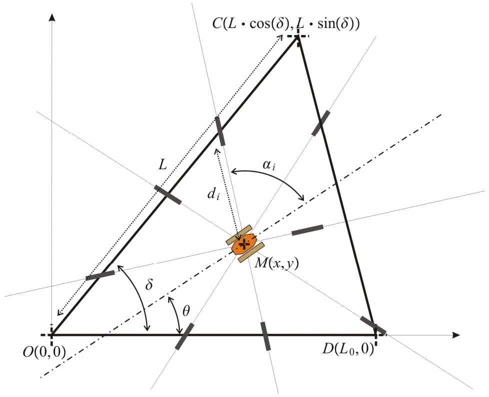
Data

- n telemetric measures di



Setting in equations

- We would like to set the SLAM problem in the form of a CSP (Constraint Satisfaction Problem) → Solve the CSP using constraint propagation methods (Interval analysis)



→

$$
\begin{cases}
\mathbf{f}(\mathbf{x}) = \mathbf{0}, \\
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\mathbf{f} : \mathbb{R}^q \to \mathbb{R}^p, \\
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\end{cases}
$$

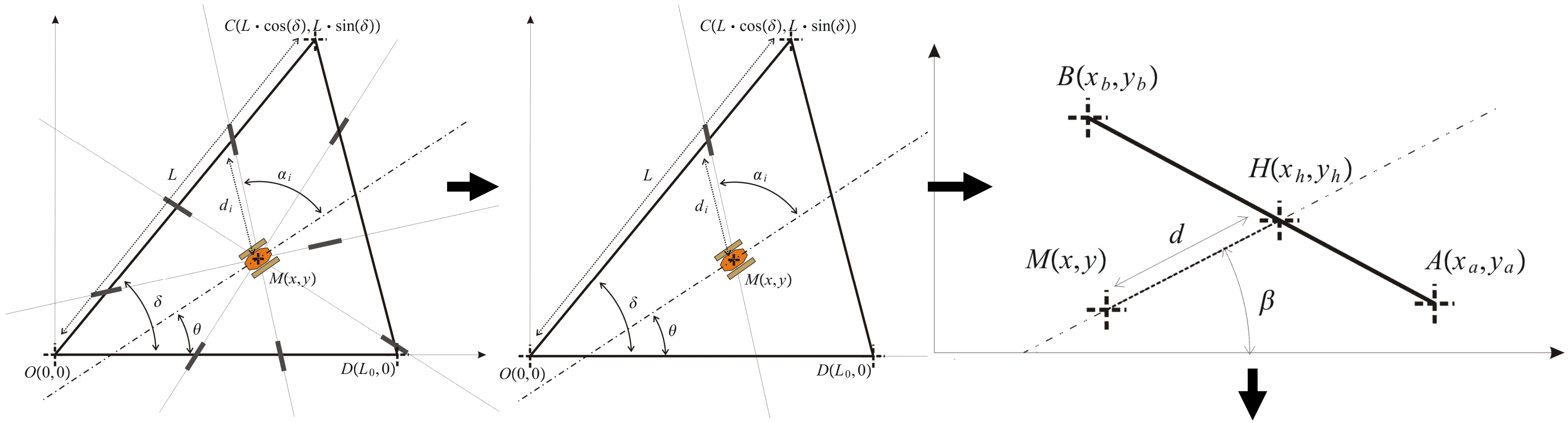
→

Interval analysis

- Good **non-linear** equation handling
- **robust** with regards to outliers

Software Demonstration in the end of the presentation

Setting in equations : calculus details



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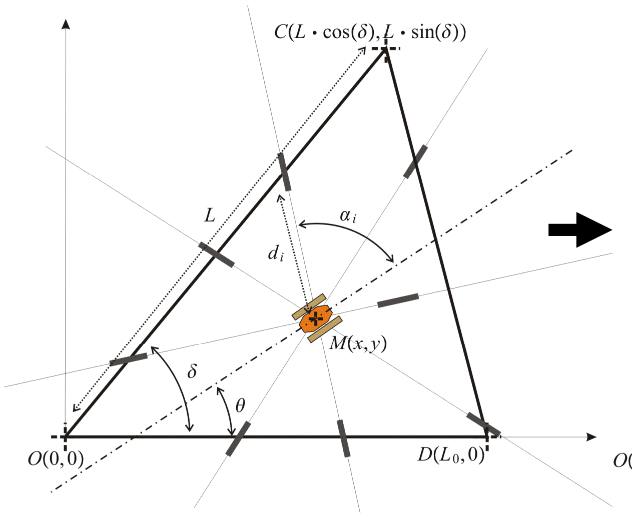

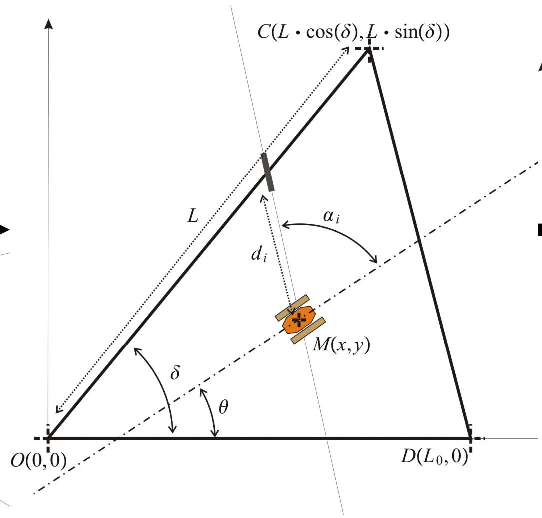

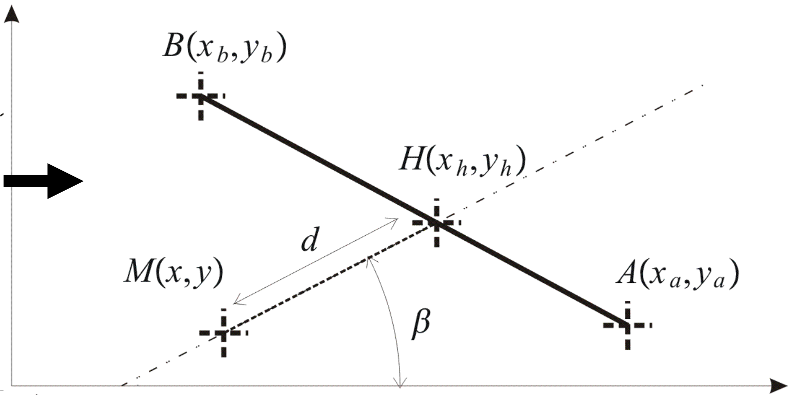

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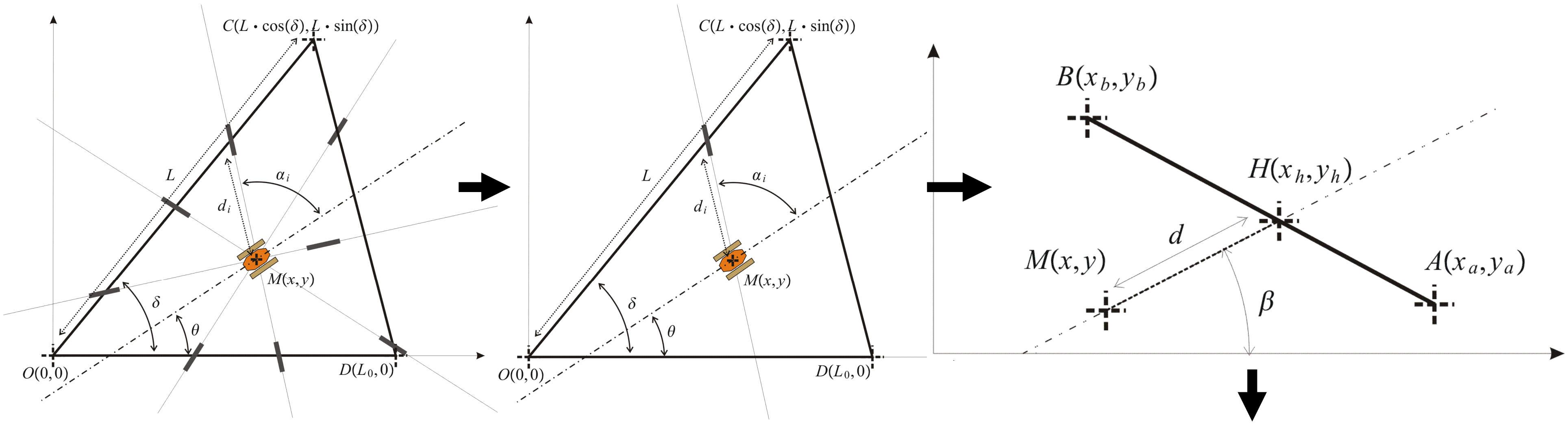

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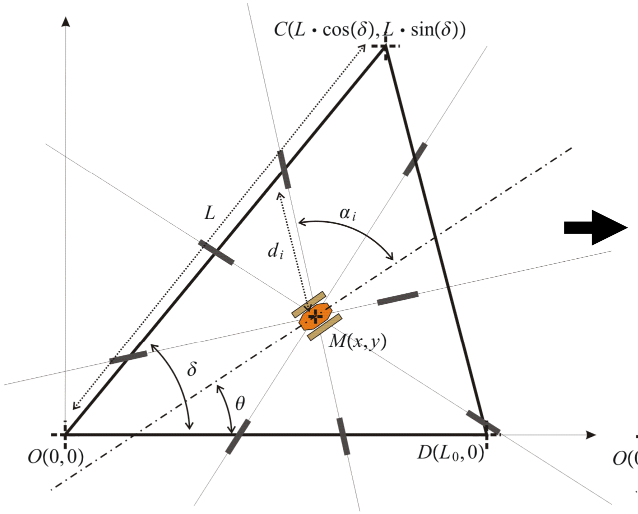

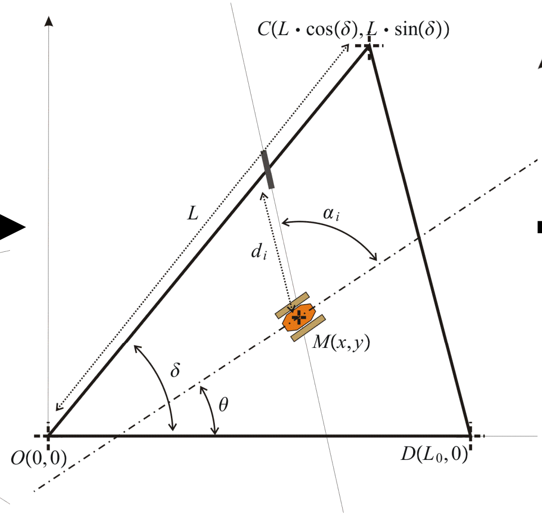

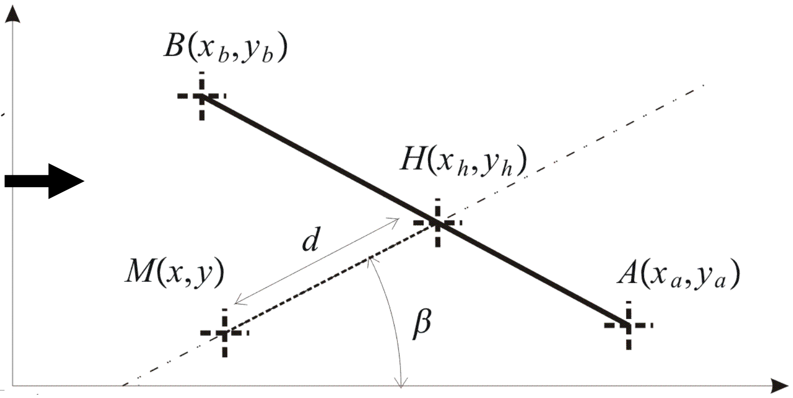

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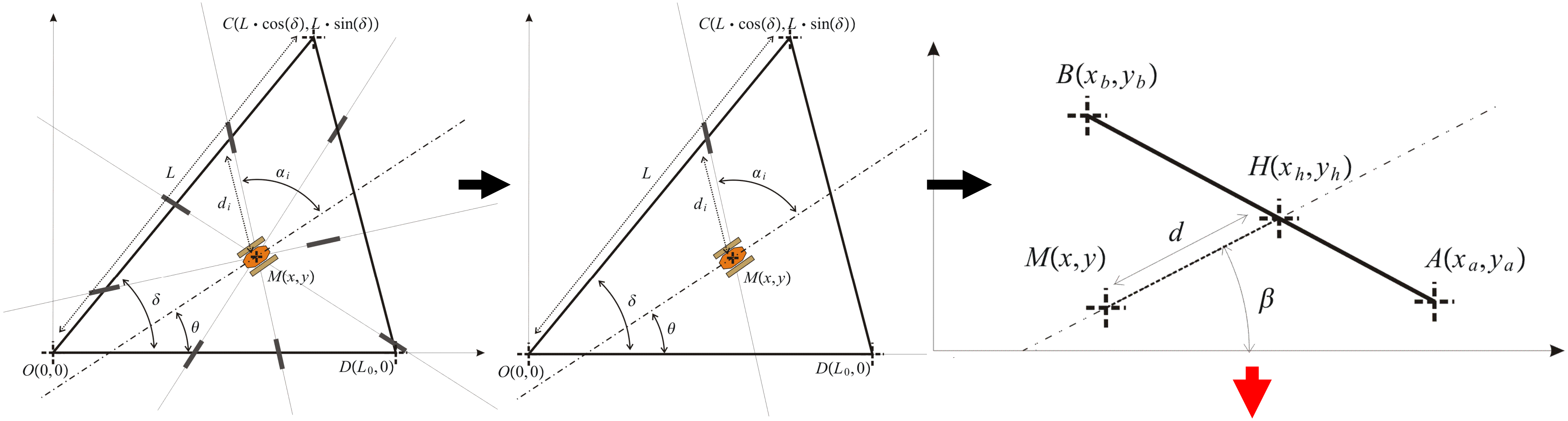

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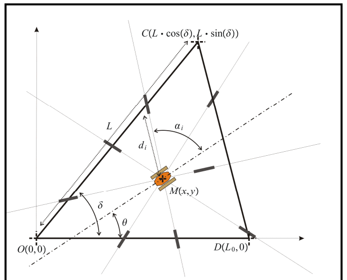

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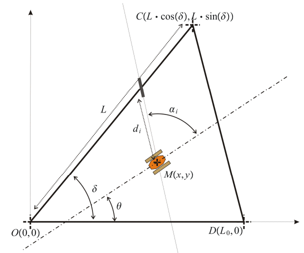

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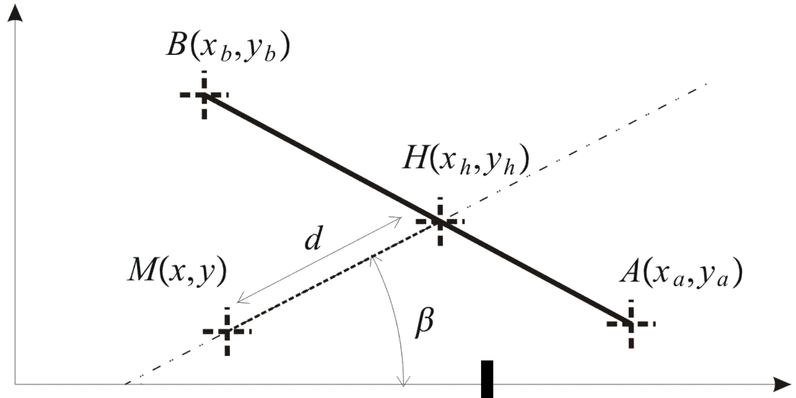

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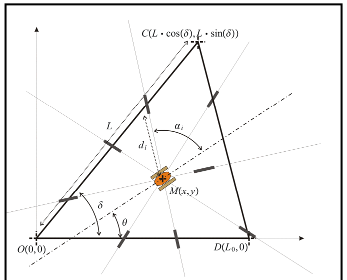

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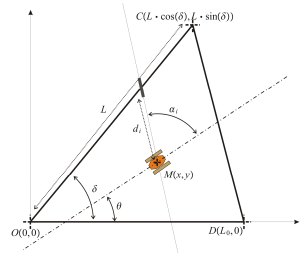

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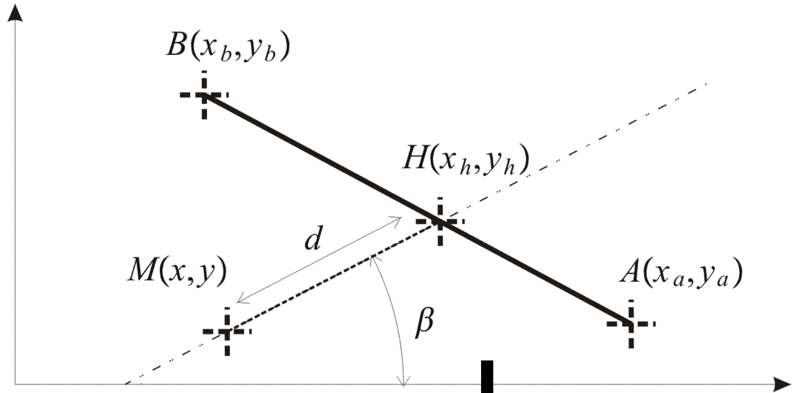

$$\det\left(\overrightarrow{AB}, \overrightarrow{AH}\right) = 0$$

$$\left\langle \overrightarrow{AH}, \overrightarrow{HB} \right\rangle > 0$$

$$\det\left(\overrightarrow{AB}, \overrightarrow{AM}\right) > 0$$

$$f_1(x, y, x_a, y_a, x_b, y_b, d, \beta) = 0$$

$$f_2(x, y, x_a, y_a, x_b, y_b, d, \beta) > 0$$

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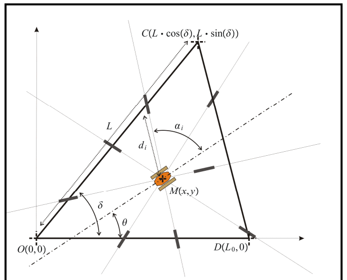

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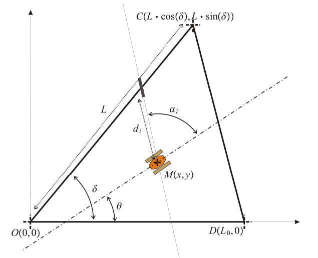

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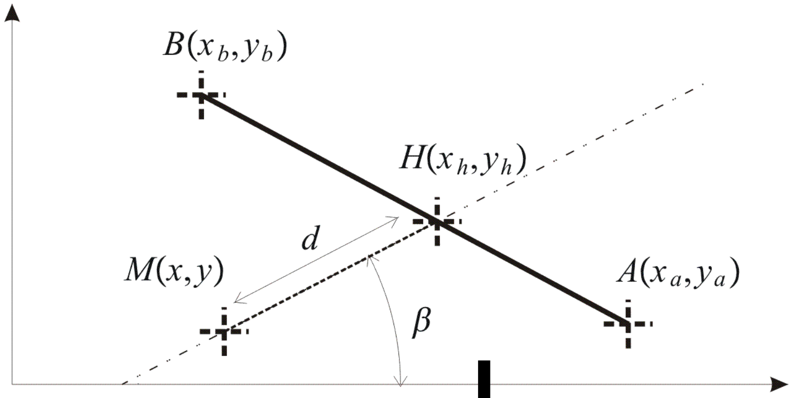

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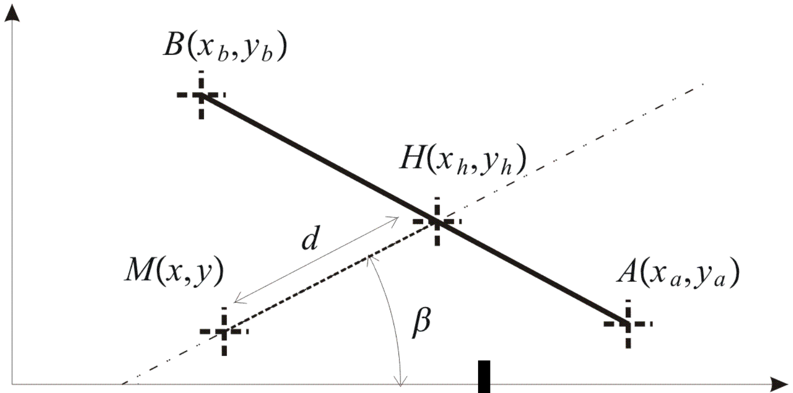

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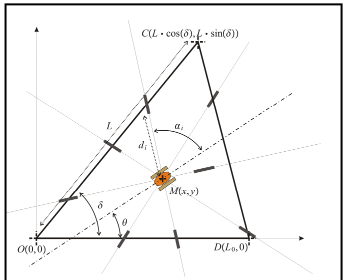

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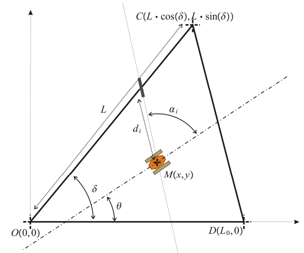

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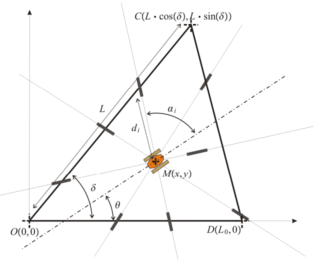

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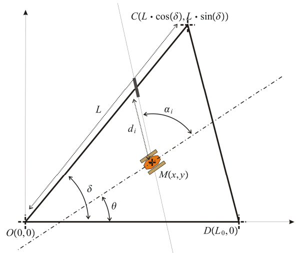

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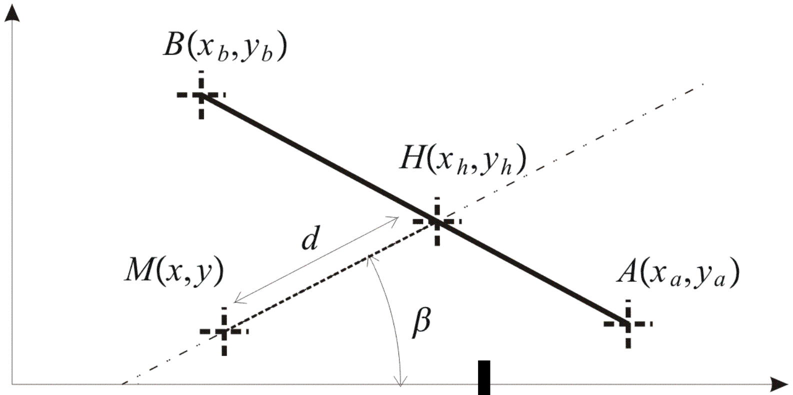

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Aparté on the AND

- The AND rule

As an example if we want

$$X_1 = 0 \textbf{ AND } X_2 = 0 \textbf{ AND } X_3 = 0$$

It is equivalent to say that

$$|X_1| + |X_2| + |X_3| = 0$$

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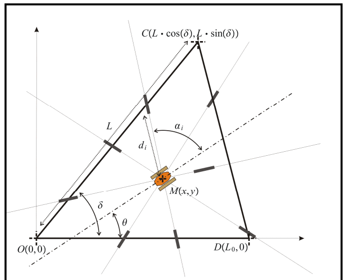

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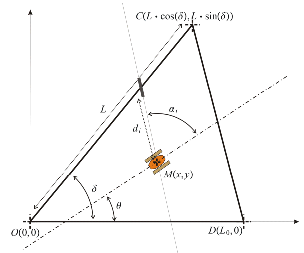

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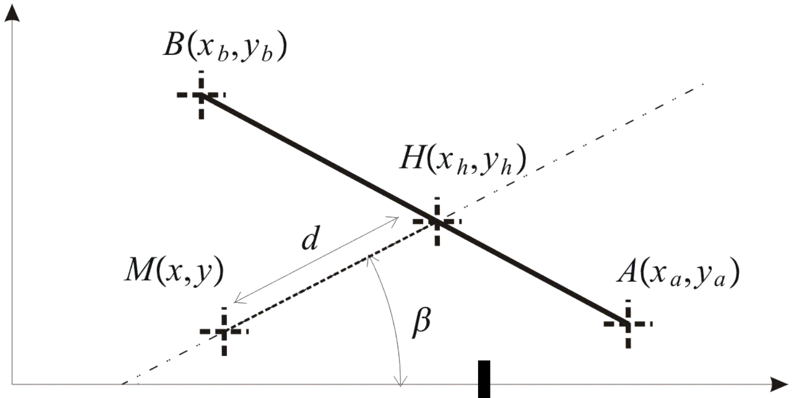

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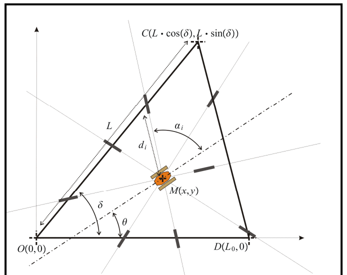

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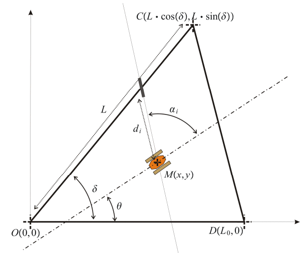

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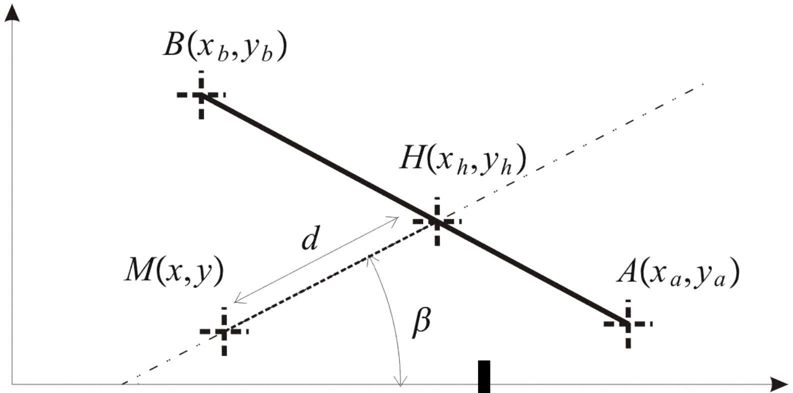


$$\det\left(\overrightarrow{AB}, \overrightarrow{AH}\right) = 0$$

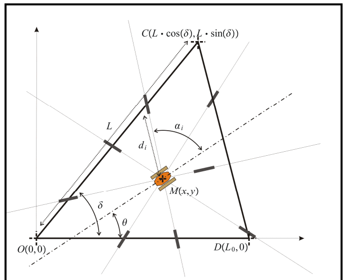
AND $\left\langle \overrightarrow{AH}, \overrightarrow{HB} \right\rangle > 0$

AND $\det\left(\overrightarrow{AB}, \overrightarrow{AM}\right) > 0$

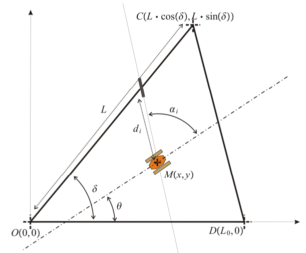
$$\begin{aligned} f_1(x,y,x_a,y_a,x_b,y_b,d,\beta) &= 0 \\ f_2(x,y,x_a,y_a,x_b,y_b,d,\beta) - z_{p2} &= 0 \\ f_3(x,y,x_a,y_a,x_b,y_b,d,\beta) - z_{p3} &= 0 \\ z_{p2} \in [0,+\infty], z_{p3} &\in [0,+\infty] \end{aligned}$$

$$\begin{cases} f_1(x,y,x_a,y_a,x_b,y_b,d,\beta) - z_1 = 0 \\ f_2(x,y,x_a,y_a,x_b,y_b,d,\beta) - z_{p2} - z_2 = 0 \\ f_3(x,y,x_a,y_a,x_b,y_b,d,\beta) - z_{p3} - z_3 = 0 \\ |z_1| + |z_2| + |z_3| - z_{ab} = 0 \\ z_{ab} = 0? \\ z_{ab} \in [-\infty,+\infty] \\ z_1 \in [-\infty,+\infty], z_2 \in [-\infty,+\infty], z_3 \in [-\infty,+\infty] \\ z_{p2} \in [0,+\infty], z_{p3} \in [0,+\infty] \end{cases}$$

Setting in equations for each **segment**

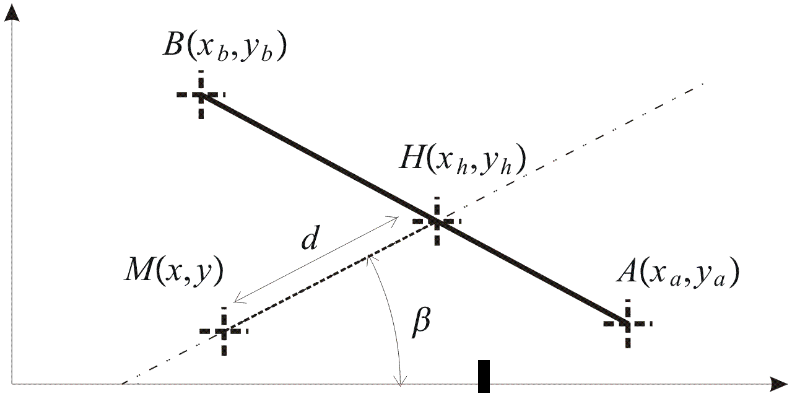


$$\begin{cases} \mathbf{f}(\mathbf{x}) = \mathbf{0}, \\ \mathbf{x} = \begin{pmatrix} \mathbf{p} \\ \mathbf{m} \\ \mathbf{v} \end{pmatrix}, \\ \mathbf{f} : \mathbb{R}^q \to \mathbb{R}^p, \\ \mathbf{x} \in [\mathbf{x}]. \end{cases}$$



$$\begin{cases} \mathbf{f}_{i^{th}\_mesure}(\mathbf{x}) = 0, \\ \mathbf{x} = \begin{pmatrix} \mathbf{p} \\ \mathbf{m} \\ \mathbf{v} \end{pmatrix}, \\ \mathbf{f}_{i^{th}\_mesure} : \mathbb{R}^q \to \mathbb{R}^m, \\ \mathbf{x} \in [\mathbf{x}]. \end{cases}$$

$$\begin{cases} \mathbf{f}_{segment\_ab}(\mathbf{x}) = 0, \\ \mathbf{x} = \begin{pmatrix} \mathbf{p} \\ \mathbf{m} \\ \mathbf{v} \end{pmatrix}, \\ \mathbf{f}_{segment\_ab} : \mathbb{R}^q \to \mathbb{R}^s, \\ \mathbf{x} \in [\mathbf{x}]. \end{cases}$$



$$\det\left(\overrightarrow{AB}, \overrightarrow{AH}\right) = 0$$

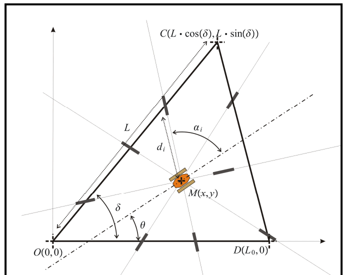
AND $\left\langle \overrightarrow{AH}, \overrightarrow{HB} \right\rangle > 0$

AND $\det\left(\overrightarrow{AB}, \overrightarrow{AM}\right) > 0$

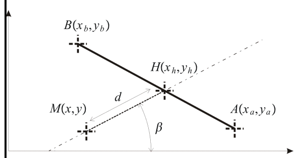
$$f_1(x, y, x_a, y_a, x_b, y_b, d, \beta) = 0$$
$$f_2(x, y, x_a, y_a, x_b, y_b, d, \beta) - z_{p2} = 0$$
$$f_3(x, y, x_a, y_a, x_b, y_b, d, \beta) - z_{p3} = 0$$
$$z_{p2} \in [0, +\infty], z_{p3} \in [0, +\infty]$$

$$\begin{cases} \mathbf{f}_{seg\_ab}(\mathbf{x}_{seg}, z_{ab}) = \mathbf{0} \\ z_{ab} = 0? \\ z_{ab} \in [-\infty, +\infty], \\ \mathbf{x}_{seg} \in [\mathbf{x}_{seg}] \end{cases}$$

Setting in equations for each **measure**

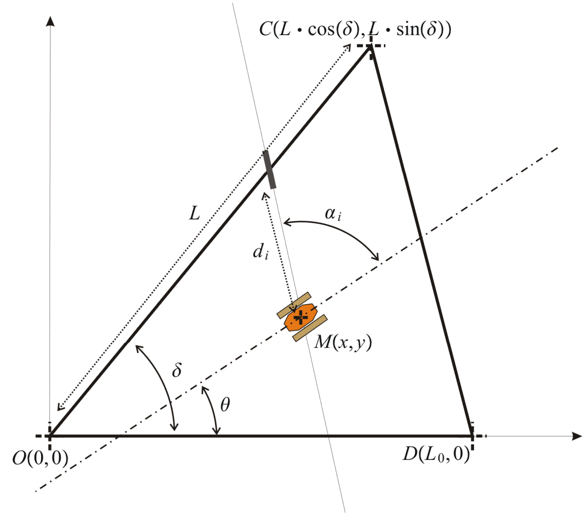


$$\begin{cases} \mathbf{f}(\mathbf{x}) = \mathbf{0}, \\ \mathbf{x} = \begin{pmatrix} \mathbf{p} \\ \mathbf{m} \\ \mathbf{v} \end{pmatrix}, \\ \mathbf{f} : \mathbb{R}^q \to \mathbb{R}^p, \\ \mathbf{x} \in [\mathbf{x}]. \end{cases}$$



$$\begin{cases} \mathbf{f}_{segment\_ab}(\mathbf{x}) = 0, \\ \mathbf{x} = \begin{pmatrix} \mathbf{p} \\ \mathbf{m} \\ \mathbf{v} \end{pmatrix}, \\ \mathbf{f}_{segment\_ab} : \mathbb{R}^q \to \mathbb{R}^s, \\ \mathbf{x} \in [\mathbf{x}]. \end{cases}$$

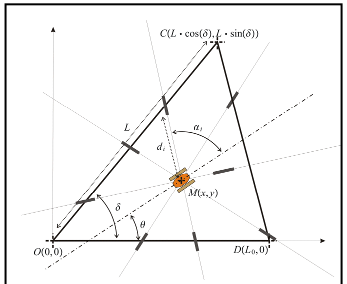
$$\begin{cases} \mathbf{f}_{i^{th}\_mesure}(\mathbf{x}) = 0, \\ \mathbf{x} = \begin{pmatrix} \mathbf{p} \\ \mathbf{m} \\ \mathbf{v} \end{pmatrix}, \\ \mathbf{f}_{i^{th}\_mesure} : \mathbb{R}^q \to \mathbb{R}^m, \\ \mathbf{x} \in [\mathbf{x}]. \end{cases}$$



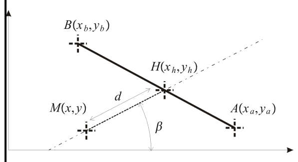
For each segment

$$\begin{cases} \mathbf{f}_{seg\_ab}(\mathbf{x}_{seg}, z_{ab}) = \mathbf{0} \\ z_{ab} = 0? \\ z_{ab} \in [-\infty, +\infty], \\ \mathbf{x}_{seg} \in [\mathbf{x}_{seg}] \end{cases}$$

Setting in equations for each **measure**

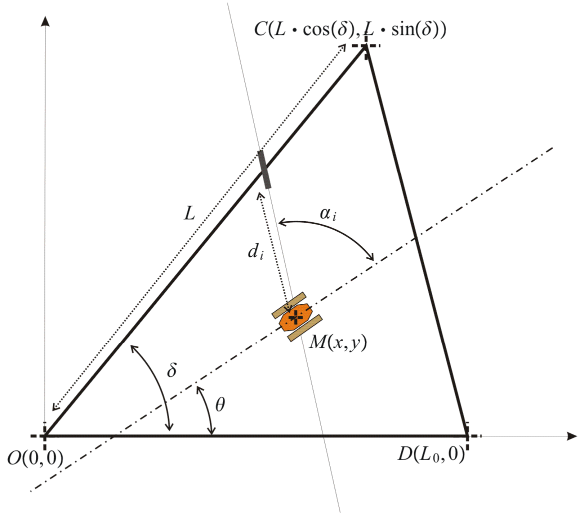


$$\begin{cases} \mathbf{f}(\mathbf{x}) = \mathbf{0}, \\ \mathbf{x} = \begin{pmatrix} \mathbf{p} \\ \mathbf{m} \\ \mathbf{v} \end{pmatrix}, \\ \mathbf{f} : \mathbb{R}^q \to \mathbb{R}^p, \\ \mathbf{x} \in [\mathbf{x}]. \end{cases}$$



$$\begin{cases} \mathbf{f}_{segment\_ab}(\mathbf{x}) = 0, \\ \mathbf{x} = \begin{pmatrix} \mathbf{p} \\ \mathbf{m} \\ \mathbf{v} \end{pmatrix}, \\ \mathbf{f}_{segment\_ab} : \mathbb{R}^q \to \mathbb{R}^s, \\ \mathbf{x} \in [\mathbf{x}]. \end{cases}$$

$$\begin{cases} \mathbf{f}_{i^{th}\_mesure}(\mathbf{x}) = 0, \\ \mathbf{x} = \begin{pmatrix} \mathbf{p} \\ \mathbf{m} \\ \mathbf{v} \end{pmatrix}, \\ \mathbf{f}_{i^{th}\_mesure} : \mathbb{R}^q \to \mathbb{R}^m, \\ \mathbf{x} \in [\mathbf{x}]. \end{cases}$$



For each segment

OR

$$\begin{cases} \mathbf{f}_{seg\_ab}(\mathbf{x}_{seg}, z_{ab}) = \mathbf{0} \\ z_{ab} = 0? \\ z_{ab} \in [-\infty, +\infty], \\ \mathbf{x}_{seg} \in [\mathbf{x}_{seg}] \end{cases}$$

Aparté on the OR

- The OR rule

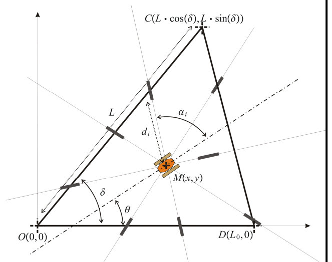
As an example if we want

$$X_1 = 0 \textbf{ OR } X_2 = 0 \textbf{ OR } X_3 = 0$$

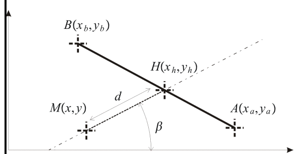
It is equivalent to say that

$$X_1 \bullet X_2 \bullet X_3 = 0$$

Setting in equations for each **measure**

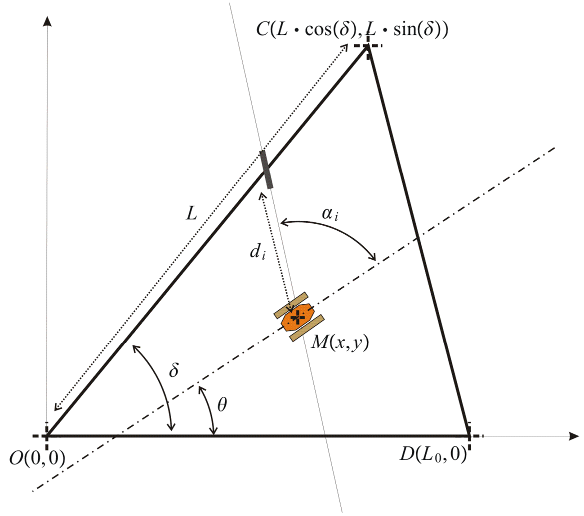


$$\begin{cases} \mathbf{f}(\mathbf{x}) = \mathbf{0}, \\ \mathbf{x} = \begin{pmatrix} \mathbf{p} \\ \mathbf{m} \\ \mathbf{v} \end{pmatrix}, \\ \mathbf{f} : \mathbb{R}^q \rightarrow \mathbb{R}^p, \\ \mathbf{x} \in [\mathbf{x}]. \end{cases}$$



$$\begin{cases} \mathbf{f}_{segment\_ab}(\mathbf{x}) = 0, \\ \mathbf{x} = \begin{pmatrix} \mathbf{p} \\ \mathbf{m} \\ \mathbf{v} \end{pmatrix}, \\ \mathbf{f}_{segment\_ab} : \mathbb{R}^q \rightarrow \mathbb{R}^s, \\ \mathbf{x} \in [\mathbf{x}]. \end{cases}$$

$$\begin{cases} \mathbf{f}_{i^{th}\_mesure}(\mathbf{x}) = 0, \\ \mathbf{x} = \begin{pmatrix} \mathbf{p} \\ \mathbf{m} \\ \mathbf{v} \end{pmatrix}, \\ \mathbf{f}_{i^{th}\_mesure} : \mathbb{R}^q \rightarrow \mathbb{R}^m, \\ \mathbf{x} \in [\mathbf{x}]. \end{cases}$$

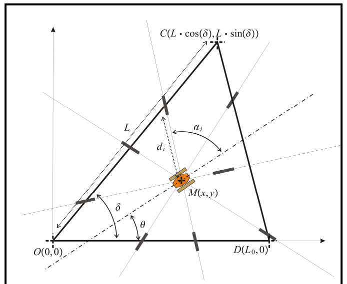


For each segment

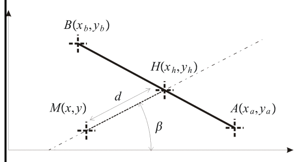
OR

$$\begin{cases} \mathbf{f}_{seg\_ab}(\mathbf{x}_{seg}, z_{ab}) = \mathbf{0} \\ z_{ab} = 0? \\ z_{ab} \in [-\infty, +\infty], \\ \mathbf{x}_{seg} \in [\mathbf{x}_{seg}] \end{cases}$$

Setting in equations for each **measure**

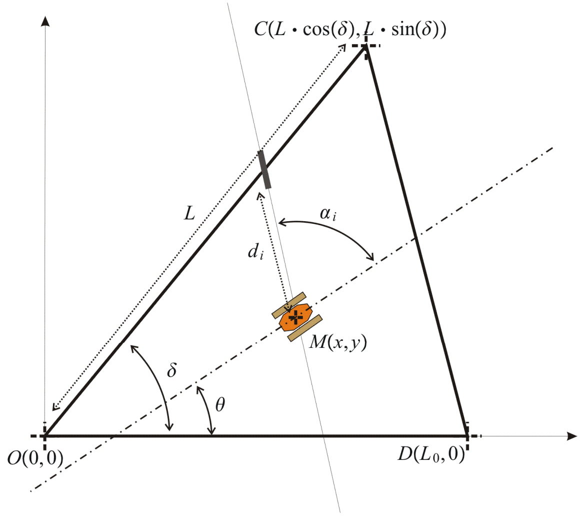


$$\begin{cases} \mathbf{f}(\mathbf{x}) = \mathbf{0}, \\ \mathbf{x} = \begin{pmatrix} \mathbf{p} \\ \mathbf{m} \\ \mathbf{v} \end{pmatrix}, \\ \mathbf{f} : \mathbb{R}^q \rightarrow \mathbb{R}^p, \\ \mathbf{x} \in [\mathbf{x}]. \end{cases}$$



$$\begin{cases} \mathbf{f}_{segment\_ab}(\mathbf{x}) = 0, \\ \mathbf{x} = \begin{pmatrix} \mathbf{p} \\ \mathbf{m} \\ \mathbf{v} \end{pmatrix}, \\ \mathbf{f}_{segment\_ab} : \mathbb{R}^q \rightarrow \mathbb{R}^s, \\ \mathbf{x} \in [\mathbf{x}]. \end{cases}$$

$$\begin{cases} \mathbf{f}_{i^{th}\_mesure}(\mathbf{x}) = 0, \\ \mathbf{x} = \begin{pmatrix} \mathbf{p} \\ \mathbf{m} \\ \mathbf{v} \end{pmatrix}, \\ \mathbf{f}_{i^{th}\_mesure} : \mathbb{R}^q \rightarrow \mathbb{R}^m, \\ \mathbf{x} \in [\mathbf{x}]. \end{cases}$$



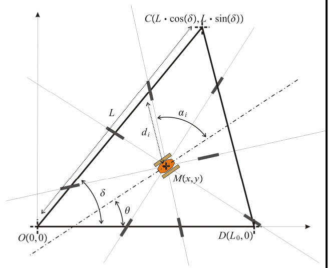
$$\begin{cases} \mathbf{f}_{iseg\_od}(\mathbf{x}, z_{iod}) = \mathbf{0} \\ \mathbf{f}_{iseg\_dc}(\mathbf{x}, z_{idc}) = \mathbf{0} \\ \mathbf{f}_{iseg\_co}(\mathbf{x}, z_{ico}) = \mathbf{0} \\ z_{iod} \cdot z_{idc} \cdot z_{ico} - z_i = 0 \\ z_i = 0? \\ z_i \in [-\infty, +\infty] \\ z_{iod} \in [-\infty, +\infty], z_{idc} \in [-\infty, +\infty], z_{ico} \in [-\infty, +\infty] \end{cases}$$

OR

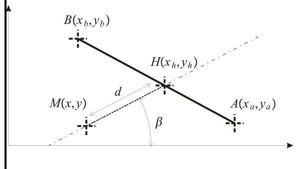
For each segment

$$\begin{cases} \mathbf{f}_{seg\_ab}(\mathbf{x}, z_{ab}) = \mathbf{0} \\ z_{ab} = 0? \\ z_{ab} \in [-\infty, +\infty], \\ \mathbf{x} \in [\mathbf{x}] \end{cases}$$

Setting in equations for each **measure**

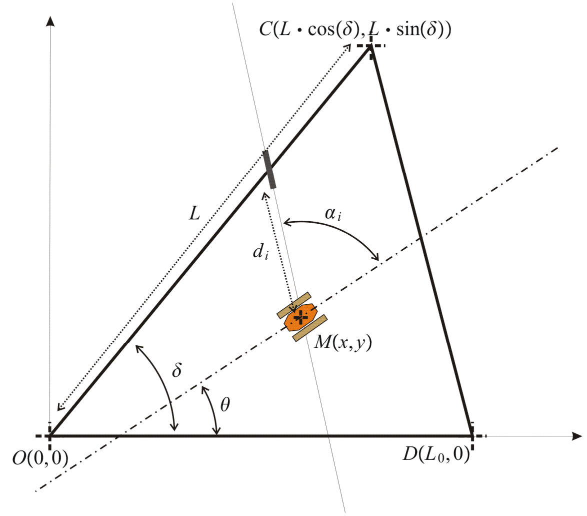


$$\begin{cases} \mathbf{f}(\mathbf{x}) = \mathbf{0}, \\ \mathbf{x} = \begin{pmatrix} \mathbf{p} \\ \mathbf{m} \\ \mathbf{v} \end{pmatrix}, \\ \mathbf{f} : \mathbb{R}^q \to \mathbb{R}^p, \\ \mathbf{x} \in [\mathbf{x}]. \end{cases}$$



$$\begin{cases} \mathbf{f}_{segment\_ab}(\mathbf{x}) = 0, \\ \mathbf{x} = \begin{pmatrix} \mathbf{p} \\ \mathbf{m} \\ \mathbf{v} \end{pmatrix}, \\ \mathbf{f}_{segment\_ab} : \mathbb{R}^q \to \mathbb{R}^s, \\ \mathbf{x} \in [\mathbf{x}]. \end{cases}$$

$$\begin{cases} \mathbf{f}_{i^{th}\_mesure}(\mathbf{x}) = 0, \\ \mathbf{x} = \begin{pmatrix} \mathbf{p} \\ \mathbf{m} \\ \mathbf{v} \end{pmatrix}, \\ \mathbf{f}_{i^{th}\_mesure} : \mathbb{R}^q \to \mathbb{R}^m, \\ \mathbf{x} \in [\mathbf{x}]. \end{cases}$$

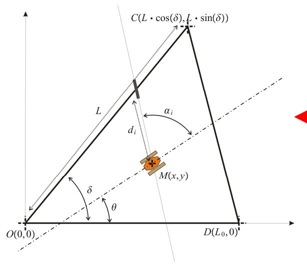


$$\begin{cases} \mathbf{f_i}(\mathbf{x}, z_i) = \mathbf{0} \\ z_i = 0? \\ z_i \in [-\infty, +\infty] \\ \mathbf{x} \in [\mathbf{x}] \end{cases}$$

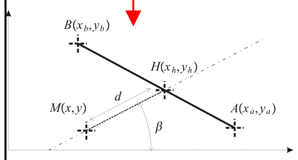
For each segment

$$\begin{cases} \mathbf{f}_{seg\_ab}(\mathbf{x}, z_{ab}) = \mathbf{0} \\ z_{ab} = 0? \\ z_{ab} \in [-\infty, +\infty], \\ \mathbf{x} \in [\mathbf{x}] \end{cases}$$

Setting in the **final** equations

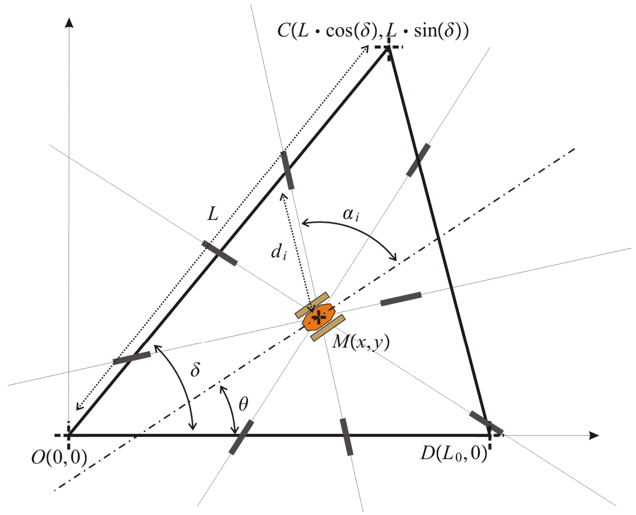


$$\begin{cases} \mathbf{f}_{i^{th}\_mesure}(\mathbf{x}) = 0, \\ \mathbf{x} = \begin{pmatrix} \mathbf{p} \\ \mathbf{m} \\ \mathbf{v} \end{pmatrix}, \\ \mathbf{f}_{i^{th}\_mesure} : \mathbb{R}^q \to \mathbb{R}^m, \\ \mathbf{x} \in [\mathbf{x}]. \end{cases}$$



$$\begin{cases} \mathbf{f}_{segment\_ab}(\mathbf{x}) = 0, \\ \mathbf{x} = \begin{pmatrix} \mathbf{p} \\ \mathbf{m} \\ \mathbf{v} \end{pmatrix}, \\ \mathbf{f}_{segment\_ab} : \mathbb{R}^q \to \mathbb{R}^s, \\ \mathbf{x} \in [\mathbf{x}]. \end{cases}$$

$$\begin{cases} \mathbf{f}(\mathbf{x}) = \mathbf{0}, \\ \mathbf{x} = \begin{pmatrix} \mathbf{p} \\ \mathbf{m} \\ \mathbf{v} \end{pmatrix}, \\ \mathbf{f} : \mathbb{R}^q \to \mathbb{R}^p, \\ \mathbf{x} \in [\mathbf{x}]. \end{cases}$$

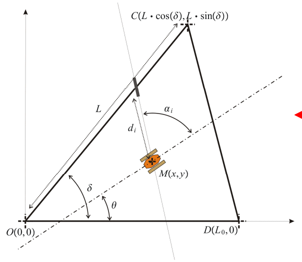


For each measure

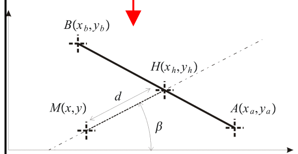
Without outliers

$$\begin{cases} \mathbf{f}_{\mathbf{i}}(\mathbf{x}, z_i) = \mathbf{0} \\ z_i = 0? \\ z_i \in [-\infty, +\infty] \\ \mathbf{x} \in [\mathbf{x}] \end{cases}$$

Setting in the **final** equations

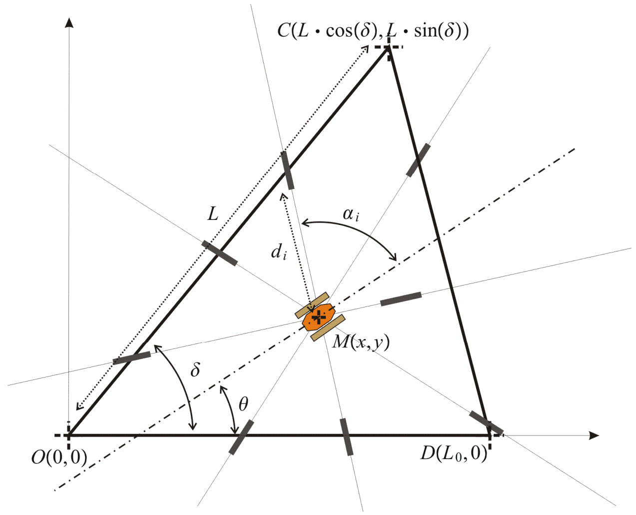


$$\begin{cases} \mathbf{f}_{i^{th}\_mesure}(\mathbf{x}) = 0, \\ \mathbf{x} = \begin{pmatrix} \mathbf{p} \\ \mathbf{m} \\ \mathbf{v} \end{pmatrix}, \\ \mathbf{f}_{i^{th}\_mesure} : \mathbb{R}^q \rightarrow \mathbb{R}^m, \\ \mathbf{x} \in [\mathbf{x}]. \end{cases}$$



$$\begin{cases} \mathbf{f}_{segment\_ab}(\mathbf{x}) = 0, \\ \mathbf{x} = \begin{pmatrix} \mathbf{p} \\ \mathbf{m} \\ \mathbf{v} \end{pmatrix}, \\ \mathbf{f}_{segment\_ab} : \mathbb{R}^q \rightarrow \mathbb{R}^s, \\ \mathbf{x} \in [\mathbf{x}]. \end{cases}$$

$$\begin{cases} \mathbf{f}(\mathbf{x}) = \mathbf{0}, \\ \mathbf{x} = \begin{pmatrix} \mathbf{p} \\ \mathbf{m} \\ \mathbf{v} \end{pmatrix}, \\ \mathbf{f} : \mathbb{R}^q \rightarrow \mathbb{R}^p, \\ \mathbf{x} \in [\mathbf{x}]. \end{cases}$$



$$\begin{cases} \mathbf{f_1}(\mathbf{x}, z_i) = \mathbf{0} \\ \ldots \\ \mathbf{f_i}(\mathbf{x}, z_i) = \mathbf{0} \\ \ldots \\ \mathbf{f_n}(\mathbf{x}, z_n) = \mathbf{0} \\ |z_1| + |z_2| + \ldots + |z_n| = 0 \\ \forall i \in \{1..n\}, z_i \in [-\infty, +\infty] \\ \mathbf{x} \in [\mathbf{x}] \end{cases}$$

AND

Without outliers

$$\begin{cases} \mathbf{f_i}(\mathbf{x}, z_i) = \mathbf{0} \\ z_i = 0? \\ z_i \in [-\infty, +\infty] \\ \mathbf{x} \in [\mathbf{x}] \end{cases}$$

Aparté on symmetric polynomials

- Basic symmetric polynomials

Definition $\phi_k(X_1,..X_n) = \sum_{1\le i_0<...<i_k\le n} X_{i_1}X_{i_2}\dots X_{i_k}$

Example $\phi_0(X_1,..,X_4) = X_1 + X_2 + X_3 + X_4$ (AND)

$\phi_1(X_1,..,X_4) = X_1X_2 + X_1X_3 + X_1X_4 + X_2X_3 + X_2X_4 + X_3X_4$

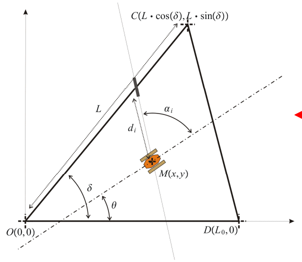
$\phi_2(X_1,..,X_4) = X_1X_2X_3 + X_1X_2X_4 + X_1X_3X_4 + X_2X_3X_4$

$\phi_3(X_1,..,X_4) = X_1X_2X_3X_4$ (OR)

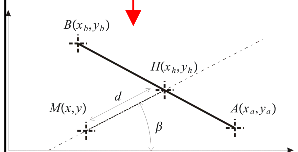
Interesting property $\forall i \in \{1,..,n\}, X_i \in [0,+\infty]$

If $\phi_\kappa(X_1,X_2,..,X_n) = 0$ then $n-\kappa$ variables among $X_1,\dots,X_n$ are equal to zero

Setting in the **final** equations

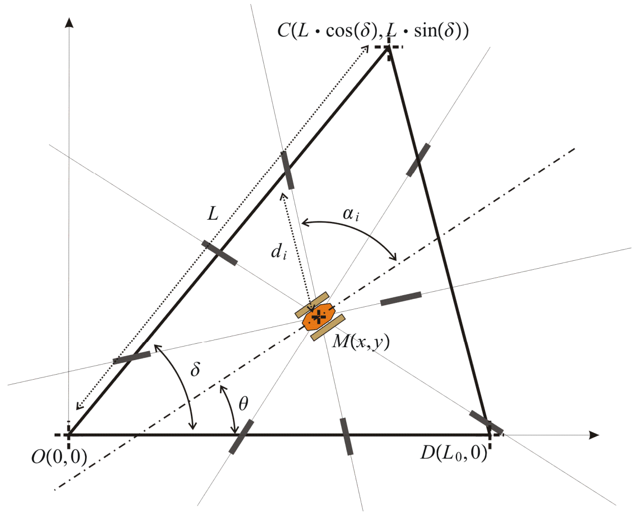


$$\begin{cases} \mathbf{f}_{i^{th}\_mesure}(\mathbf{x}) = 0, \\ \mathbf{x} = \begin{pmatrix}\mathbf{p}\\ \mathbf{m}\\ \mathbf{v}\end{pmatrix}, \\ \mathbf{f}_{i^{th}\_mesure} : \mathbb{R}^q \to \mathbb{R}^m, \\ \mathbf{x} \in [\mathbf{x}]. \end{cases}$$



$$\begin{cases} \mathbf{f}_{segment\_ab}(\mathbf{x}) = 0, \\ \mathbf{x} = \begin{pmatrix}\mathbf{p}\\ \mathbf{m}\\ \mathbf{v}\end{pmatrix}, \\ \mathbf{f}_{segment\_ab} : \mathbb{R}^q \to \mathbb{R}^s, \\ \mathbf{x} \in [\mathbf{x}]. \end{cases}$$

$$\begin{cases} \mathbf{f}(\mathbf{x}) = \mathbf{0}, \\ \mathbf{x} = \begin{pmatrix}\mathbf{p}\\ \mathbf{m}\\ \mathbf{v}\end{pmatrix}, \\ \mathbf{f} : \mathbb{R}^q \to \mathbb{R}^p, \\ \mathbf{x} \in [\mathbf{x}]. \end{cases}$$



$$\begin{cases} \mathbf{f}_1(\mathbf{x}, z_i) = \mathbf{0} \\ \dots \\ \mathbf{f}_i(\mathbf{x}, z_n) = \mathbf{0} \\ \dots \\ \mathbf{f}_n(\mathbf{x}, z_n) = \mathbf{0} \\ \phi_\kappa(|\mathbf{z}_1|, |\mathbf{z}_2|, \dots, |\mathbf{z}_n|) = \sum_{1 \le i_0 < \dots < i_\kappa \le n} |\mathbf{z}_{\mathbf{i}_1}| \cdot |\mathbf{z}_{\mathbf{i}_2}| \dots \cdot |\mathbf{z}_{\mathbf{i}_\kappa}| = 0 \\ \forall i \in \{1..n\}, z_i \in [-\infty, +\infty] \\ \mathbf{x} \in [\mathbf{x}] \end{cases}$$

k outlier

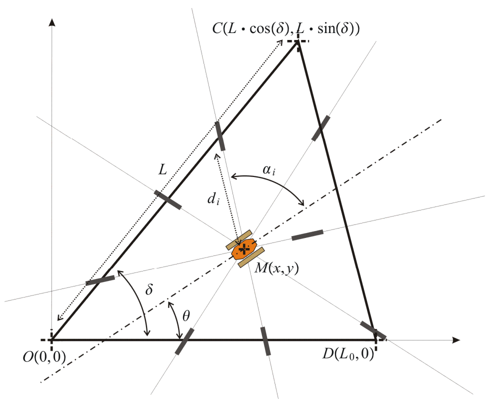
$$\begin{cases} \mathbf{f}_\mathbf{i}(\mathbf{x}, z_i) = \mathbf{0} \\ z_i = 0? \\ z_i \in [-\infty, +\infty] \\ \mathbf{x} \in [\mathbf{x}] \end{cases}$$

The final CSP

$$
\left\{
\begin{array}{l}
\ldots \\
\ldots \\
L_0 \cdot (y + d_i \cdot \sin(\theta + \alpha_i)) - z_{iod1} = 0 \\
(x + d_i \cdot \cos(\theta + \alpha_i))(L_0 - x - d_i \cdot \cos(\theta + \alpha_i)) - (y + d_i \cdot \sin(\theta + \alpha_i))^2 - z_{piod2} - z_{iod2} = 0 \\
L_0 \cdot y - z_{piod3} - z_{iod3} = 0 \\
|z_{iod1}| + |z_{iod2}| + |z_{iod3}| - z_{iod} = 0 \\
(L\sin(\delta) - L_0)(y + d_i \cdot \sin(\theta + \alpha_i)) + L\sin(\delta)(x + d_i \cdot \cos(\theta + \alpha_i) - L_0) - z_{idc1} = 0 \\
(x + d_i \cdot \cos(\theta + \alpha_i) - L_0)(L\cos(\delta) - x - d_i \cdot \cos(\theta + \alpha_i)) + (y + d_i \cdot \sin(\theta + \alpha_i))(L\sin(\delta) - y - d_i \cdot \sin(\theta + \alpha_i)) - z_{pidc2} - z_{idc2} = 0 \\
(L\cos(\delta) - L_0) \cdot y - L\sin(\delta)(x - L_0) - z_{pidc3} - z_{idc3} = 0 \\
|z_{idc1}| + |z_{idc2}| + |z_{idc3}| - z_{idc} = 0 \\
-L\cos(\delta)(y + d_i \cdot \sin(\theta + \alpha_i) - L\sin(\delta)) - L\sin(\delta)(x + d_i \cdot \cos(\theta + \alpha_i) - L\cos(\delta)) - z_{ico1} = 0 \\
(x + d_i \cdot \cos(\theta + \alpha_i) - L\cos(\delta))(-x - d_i \cdot \cos(\theta + \alpha_i)) + (y + d_i \cdot \sin(\theta + \alpha_i) - L\sin(\delta))(-y - d_i \cdot \sin(\theta + \alpha_i)) - z_{pico2} - z_{ico2} = 0 \\
-L\cos(\delta)(y - L\sin(\delta)) + L\sin(\delta)(x - L\cos(\delta)) - z_{pico3} - z_{ico3} = 0 \\
|z_{ico1}| + |z_{ico2}| + |z_{ico3}| - z_{ico} = 0 \\
z_{iod} \cdot z_{idc} \cdot z_{ico} - z_i = 0 \\
\ldots \\
\ldots \\
\phi_\kappa(|z_1|, |z_2|, \ldots, |z_n|) = \sum_{1 \le i_0 < \ldots < i_\kappa \le n} |z_{i_1}| \cdot |z_{i_2}| \ldots \cdot |z_{i_\kappa}| = 0 \\
x \in [x], y \in [y], \theta \in [\theta], L \in [L], \delta \in [\delta] \\
\forall i \in \{1..n\}, d_i \in [d_i], \{z_{i*} \ldots\} \subset [-\infty, +\infty], \{z_{pi*} \ldots\} \subset [0, +\infty]
\end{array}
\right.
$$

Software demonstration

- We would like to set the SLAM problem in the form of a CSP (Constraint Satisfaction Problem) ➞ Solve the CSP using constraint propagation methods (Interval analysis)



➞

$$
\begin{cases}
\mathbf{f}(\mathbf{x}) = \mathbf{0}, \\
\mathbf{x} = \begin{pmatrix} \mathbf{p} \\ \mathbf{m} \\ \mathbf{v} \end{pmatrix}, \\
\mathbf{f} : \mathbb{R}^q \rightarrow \mathbb{R}^p, \\
\mathbf{x} \in [\mathbf{x}].
\end{cases}
$$

➞

Interval analysis

•Good non-linear equation handling

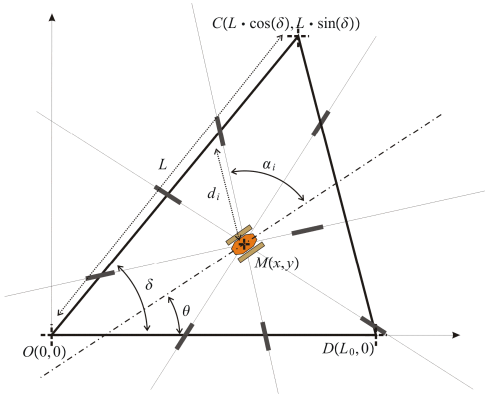
• **robust** with regards to outliers

Software Demonstration in the end of the presentation

??? Questions ???

Abstract

- We would like to set the SLAM problem in the form of a CSP (Constraint Satisfaction Problem) → Solve the CSP using constraint propagation methods (Interval analysis)



→

$$\begin{cases} \mathbf{f}(\mathbf{x}) = \mathbf{0}, \\ \mathbf{x} = \begin{pmatrix} \mathbf{p} \\ \mathbf{m} \\ \mathbf{v} \end{pmatrix}, \\ \mathbf{f} : \mathbb{R}^q \rightarrow \mathbb{R}^p, \\ \mathbf{x} \in [\mathbf{x}]. \end{cases}$$

→

Interval analysis

•Good non-linear equation handling

• **robust** with regards to outliers

Software Demonstration in the end of the presentation