

# Image derived contractor and its application in robot localization

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**“Robust solving of localization, mapping and SLAM problems in the field of underwater robotics using set membership methods”**

My website: [www.ensieta.fr/sliwka](http://www.ensieta.fr/sliwka)

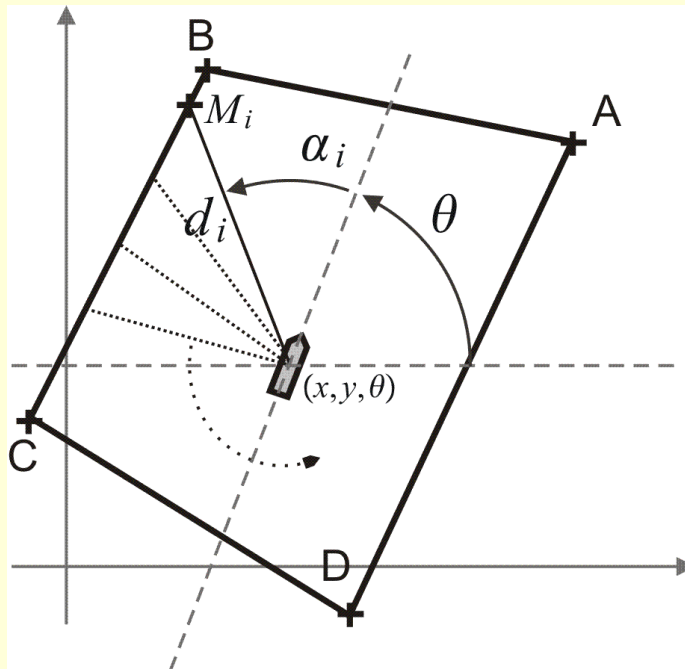
M Luc JAULIN's website: [www.ensieta.fr/jaulin](http://www.ensieta.fr/jaulin)



# Context

Localization :

- Map (structured environment, **binary image**)
- Position (X, Y, Heading)
- Measures (Sonar measures and compass)



$\forall (d_i, \alpha_i, \theta_i) \in \text{Correct\_Measures}$

$(x, y, \theta) \in \text{Pose}, \exists (x_{map}, y_{map}) \in \text{Map}$

$(x, y, \theta) = H(d_i, \alpha_i, \theta_i, x_{map}, y_{map})$

# Context



Localization :

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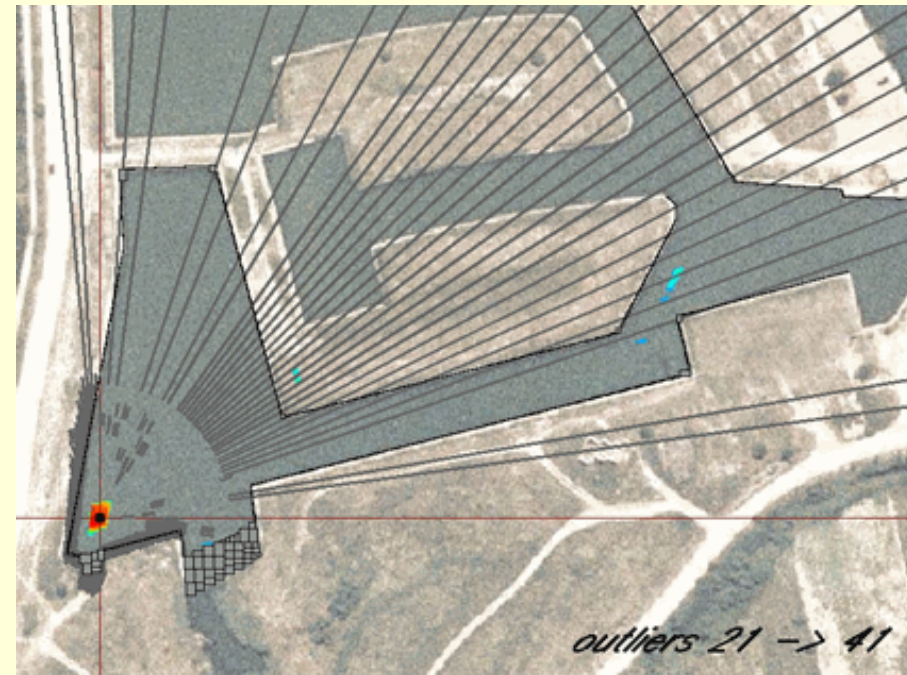


# Context



Localization :

- Map (structured environment, **binary image**)
- Position (X, Y, Heading)
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# Contractor on binary image

- **Goal :**
  - **Not** limit oneself to **structured** environments
  - Image representation : use **image processing** optimized algorithms (OpenCV)
  - Use of integers : Implement on hardware for **parallel computing** as the FPGA.

# What is an image contractor

Consider a binary image

> Continuous representation :

$$f : \mathbb{R}^2 \rightarrow \{0, 1\}$$

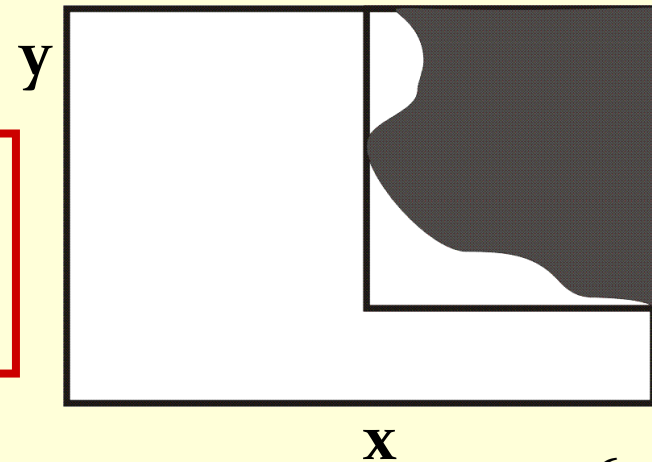
> Discrete representation

$$f : [1..Width] \times [1..Height] \rightarrow \{0, 1\}$$

Contraction of a box  $P$

$$E = \{(x, y) \in P, f(x, y) = 1\}$$

$$Contracted\_P = enclosing\_box\_of(E)$$

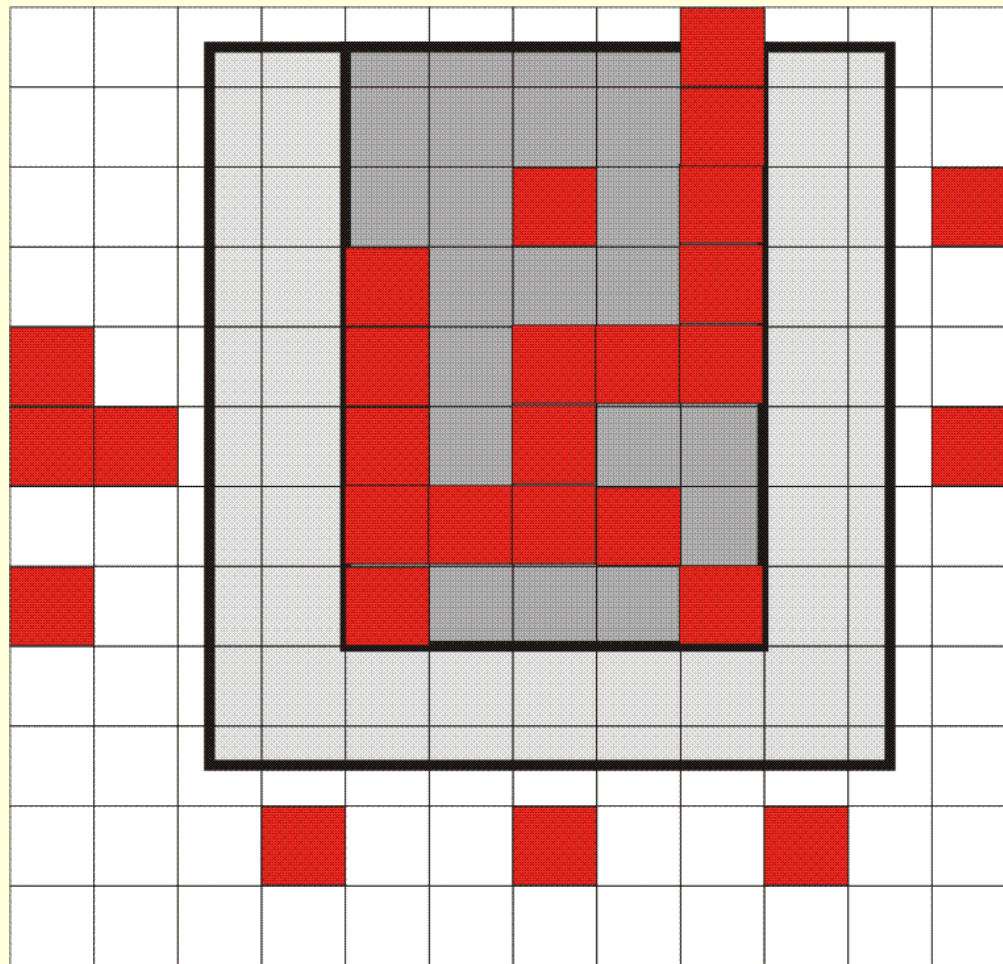


# Plan

- Show 4 methods to implement Image Contractor
- Some applications

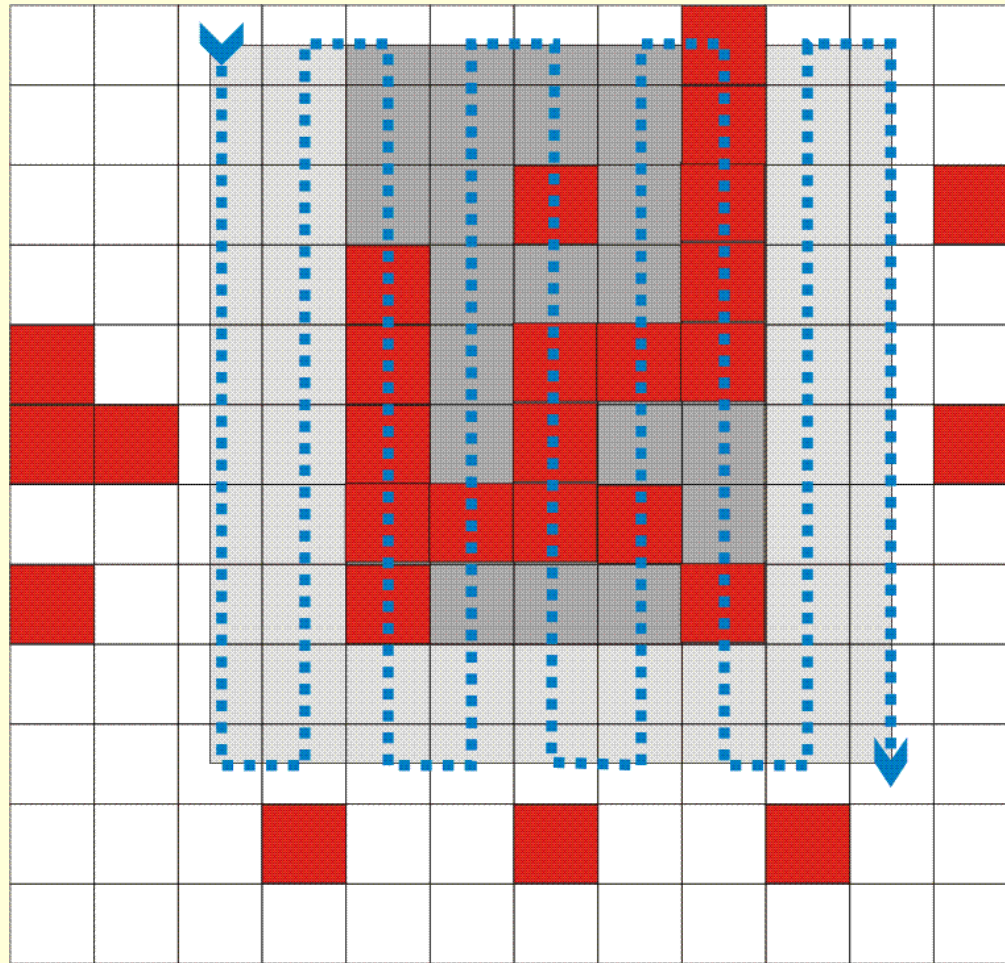


# Example

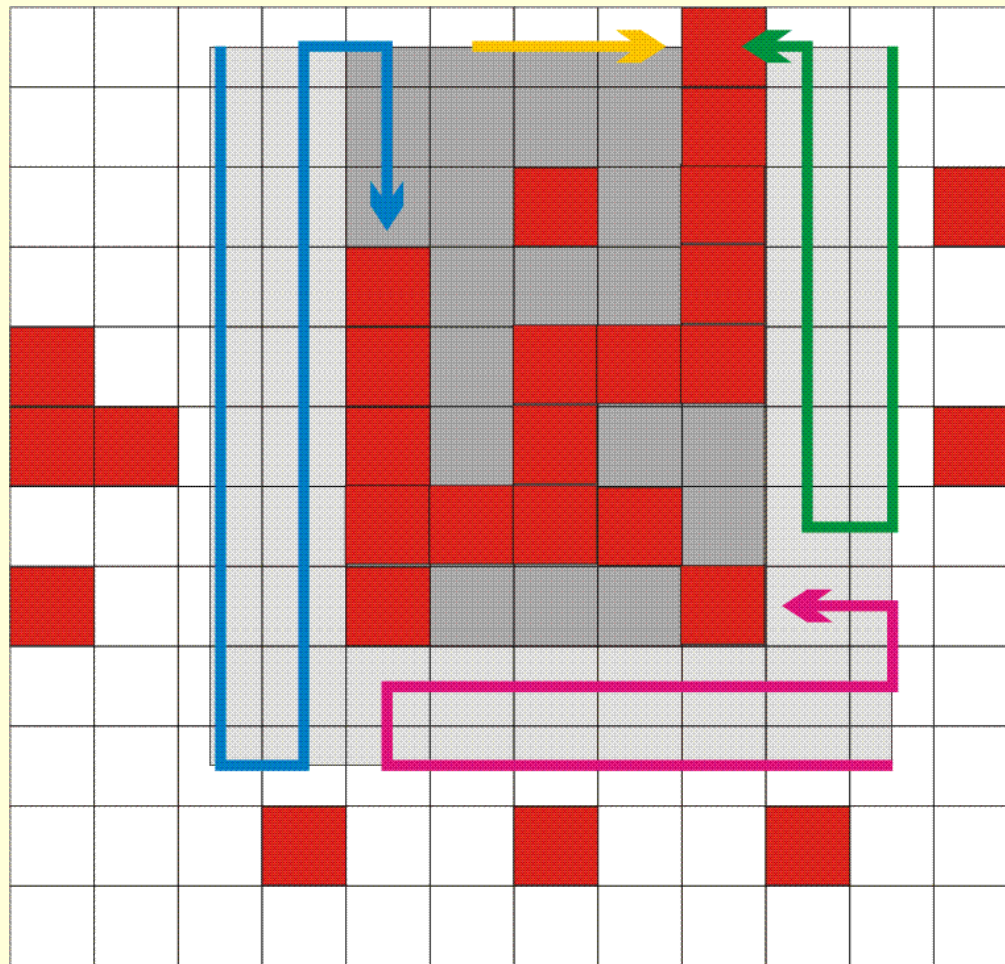




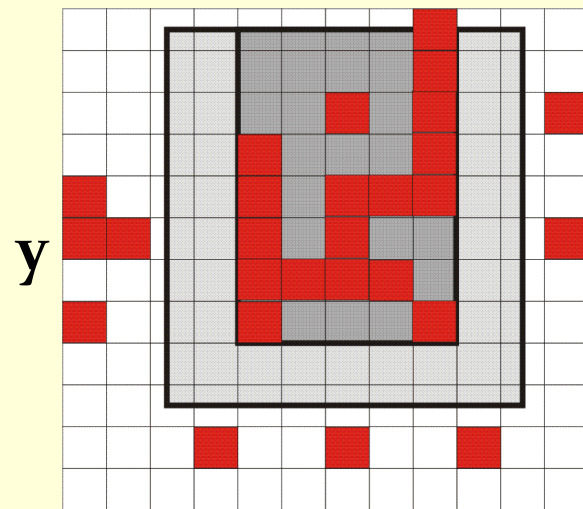
# naïve method (directly on image)



# Less naïve method (directly on image)



# Second method : 2D structure



**x**  
**Image and  
example of  
contraction**

**re-organize  
once for all**  
→

**Sort by X**

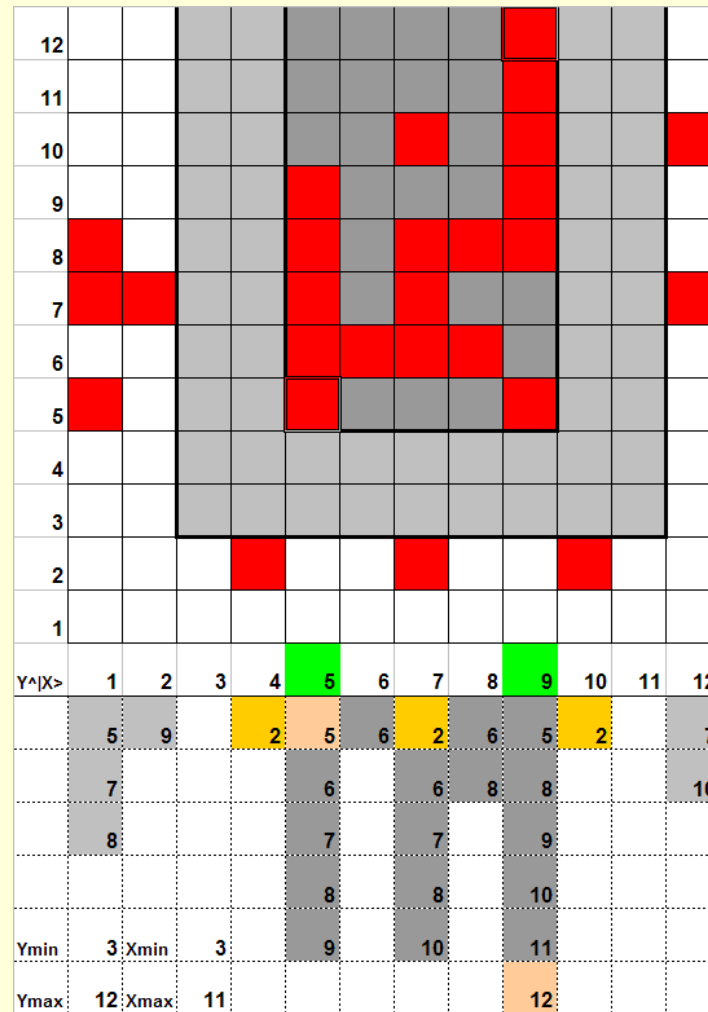
<b>1</b>	<b>2</b>		<b>n</b>
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<b>1</b>	<b>1</b>
<b>2</b>	<b>2</b>
<b>n1</b>	<b>n2</b>

**Sort by Y**

**New data structure**

# Second method : 2D structure

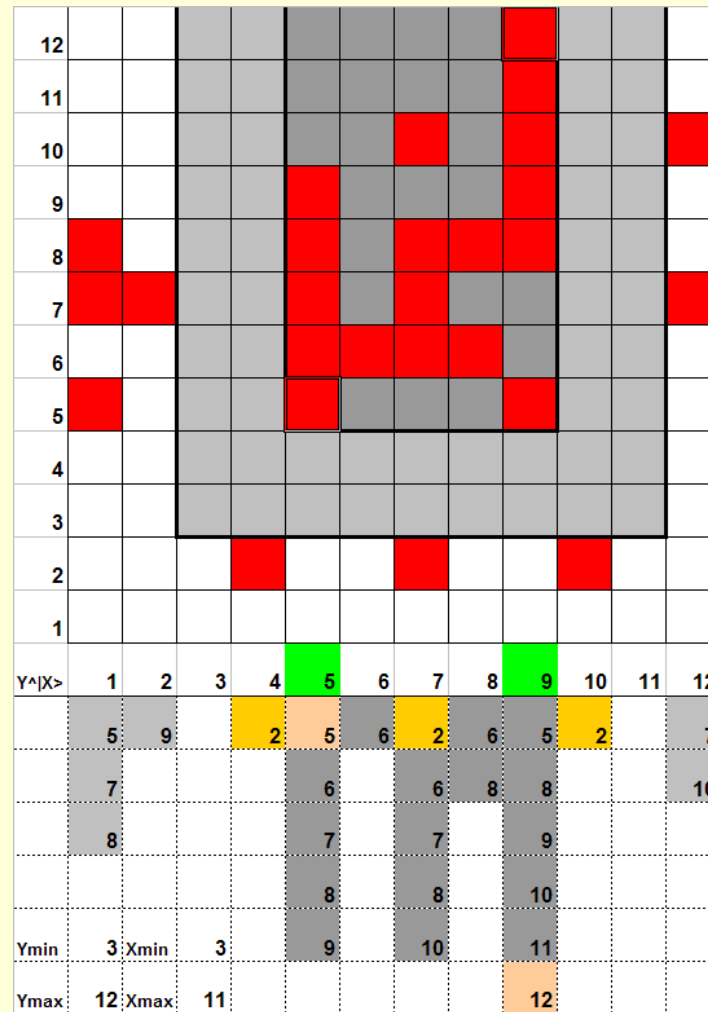




# Second method : 2D structure

4												
3												
2												
1												
$Y^{ X>$	1	2	3	4	5	6	7	8	9	10	11	12
	5	9		2	5	6	2	6	5	2		7
	7				6		6	8	8			10
	8				7		7		9			
					8		8		10			
$Y_{min}$	3	$X_{min}$	3		9		10		11			
$Y_{max}$	12	$X_{max}$	11						12			

# Second method : 2D structure



# third method : restructure to binary tree

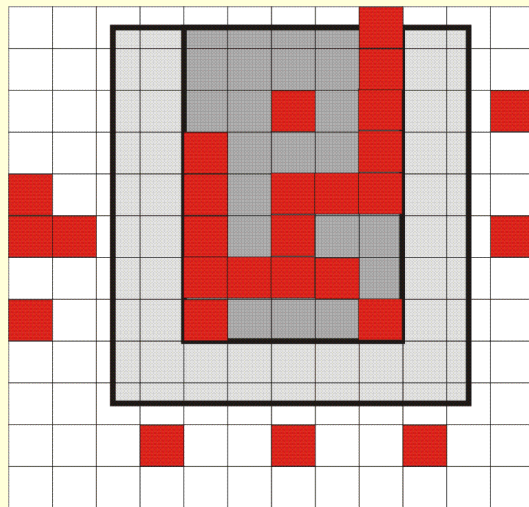
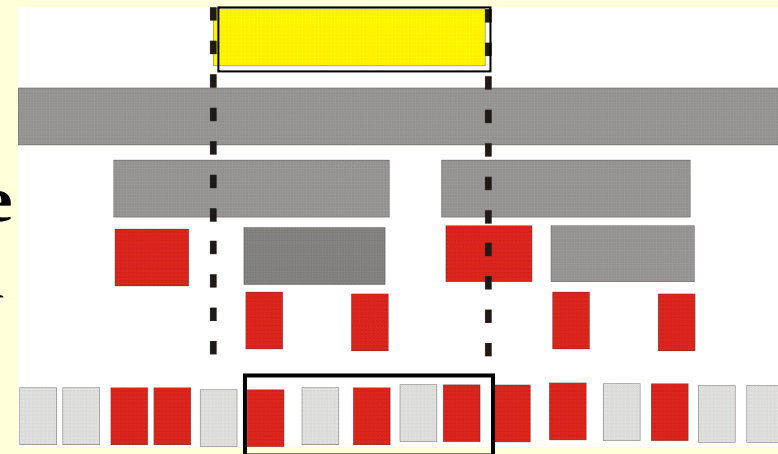


Image and example of contraction

re-organize  
once for all



Binary tree representation  
of an image and  
example of contraction



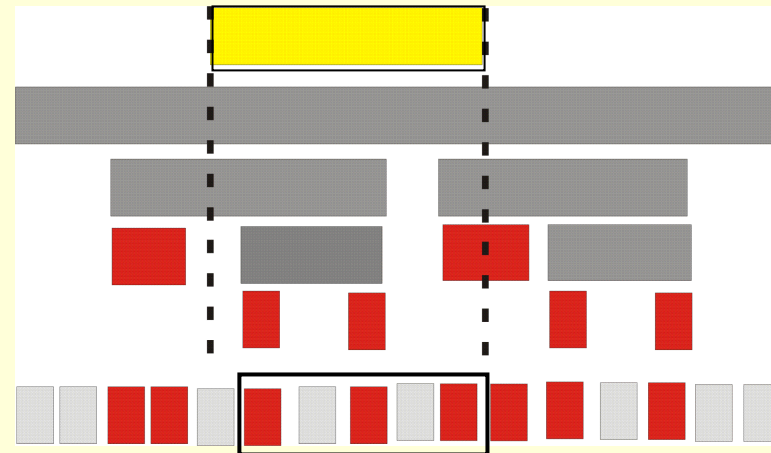


# third method : binary tree

## The algorithm

### Output : Union of boxes

- consecutive inclusion / intersection tests on nodes
- If **inclusion** true add the node / leaf to a list of boxes
- If **intersection** true :
  - If leaf add intersection to list
  - If node apply algorithm on children of the node
- If **exclusion** stop
- End : The result will be the union of those boxes



## Binary tree representation of an image and example of contraction

## Fourth method :

Consider a binary image

> Continuous representation for easier explanation :

$$f : \mathbb{R}^2 \rightarrow \{0, 1\}$$

Consider the function

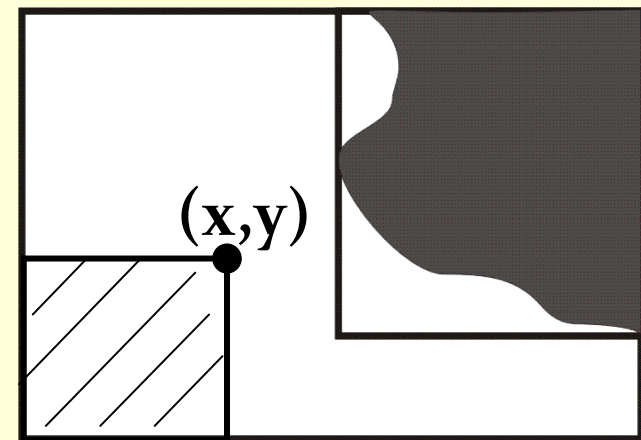
$$\psi : \mathbb{R}^2 \rightarrow \mathbb{R}$$

$$\psi(x, y) = \int_0^x \int_0^y f(x, y) dx dy$$



Number of 1-pixels in the box

$$[0, x] \times [0, y]$$



## Fourth method :

Consider a binary image

$$f : \mathbb{R}^2 \rightarrow \{0, 1\}$$

Consider the function

$$\phi : [\mathbb{R}]^2 \rightarrow \mathbb{R},$$

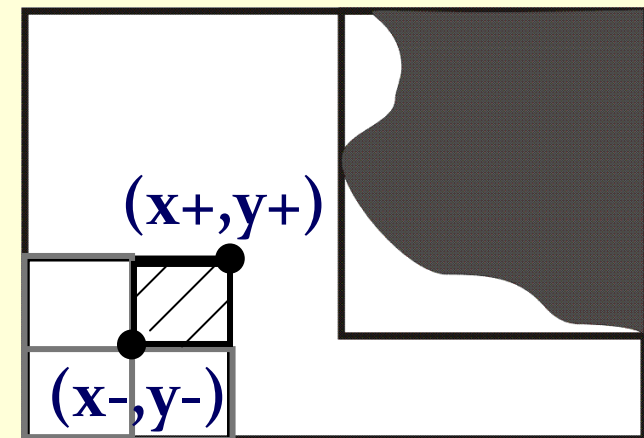
$$\phi([x], [y]) = \int_{(x,y) \in [x][y]} f(x,y) dx dy$$

$$[x] = [x^-, x^+], [y] = [y^-, y^+]$$

$$\begin{aligned} \phi([x], [y]) &= \psi(x^+, y^+) - \psi(x^-, y^+) \\ &\quad - \psi(x^+, y^-) + \psi(x^-, y^-) \end{aligned}$$

Number of 1-pixels in the box

$$[x^-, x^+] \times [y^-, y^+]$$



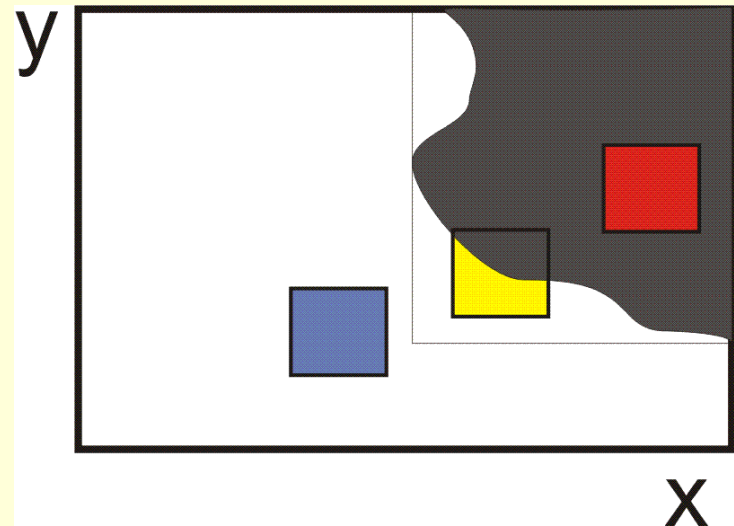
## Fourth method :

Consider a binary image  $f : \mathbb{R}^2 \rightarrow \{0, 1\}$

Compute  $\Psi$  Once!

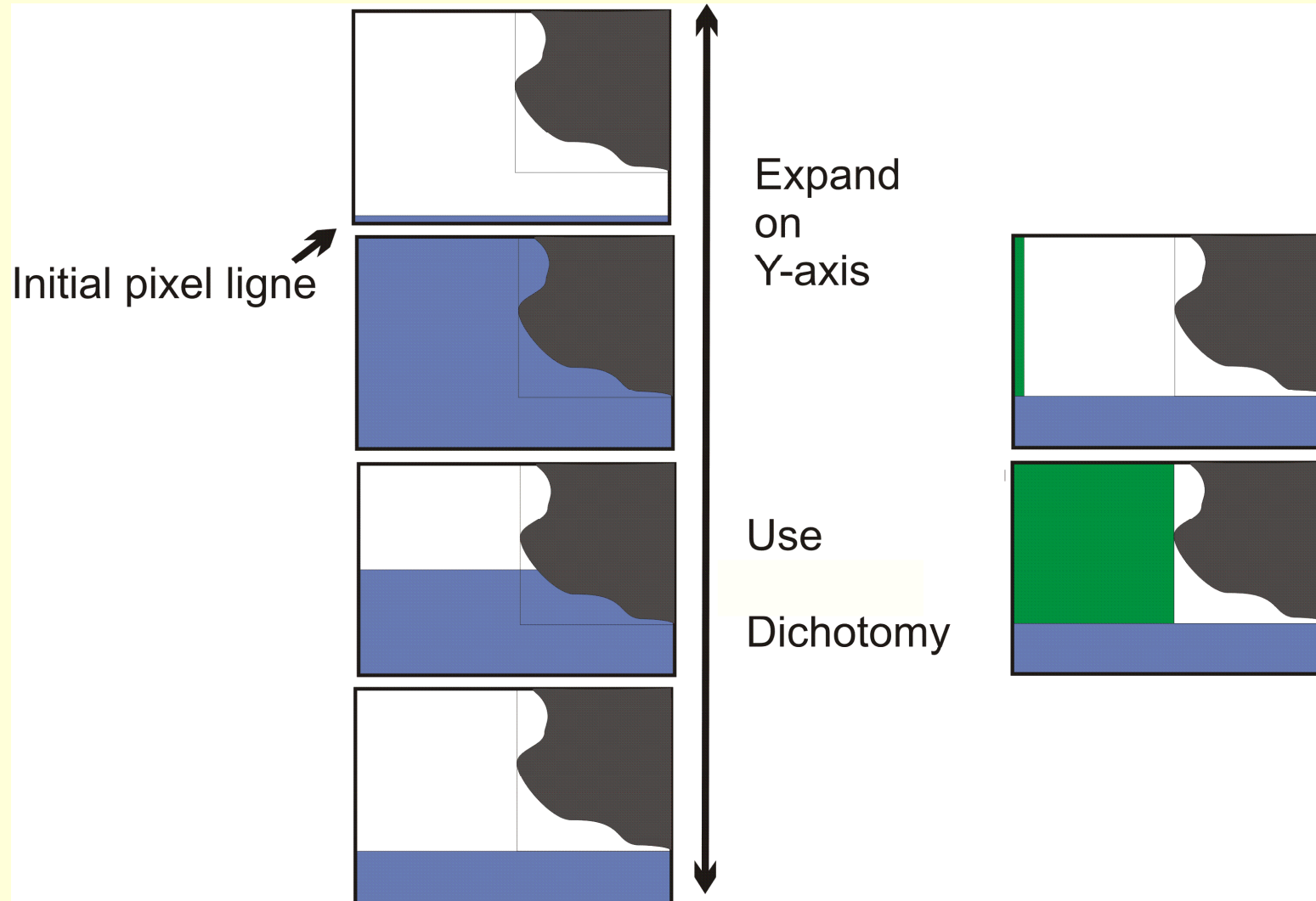
$$\phi([x], [y]) = \psi(x^+, y^+) - \psi(x^-, y^+) - \psi(x^+, y^-) + \psi(x^-, y^-)$$

Number of 1-pixels in the box  
 $[x^-, x^+] \times [y^-, y^+]$

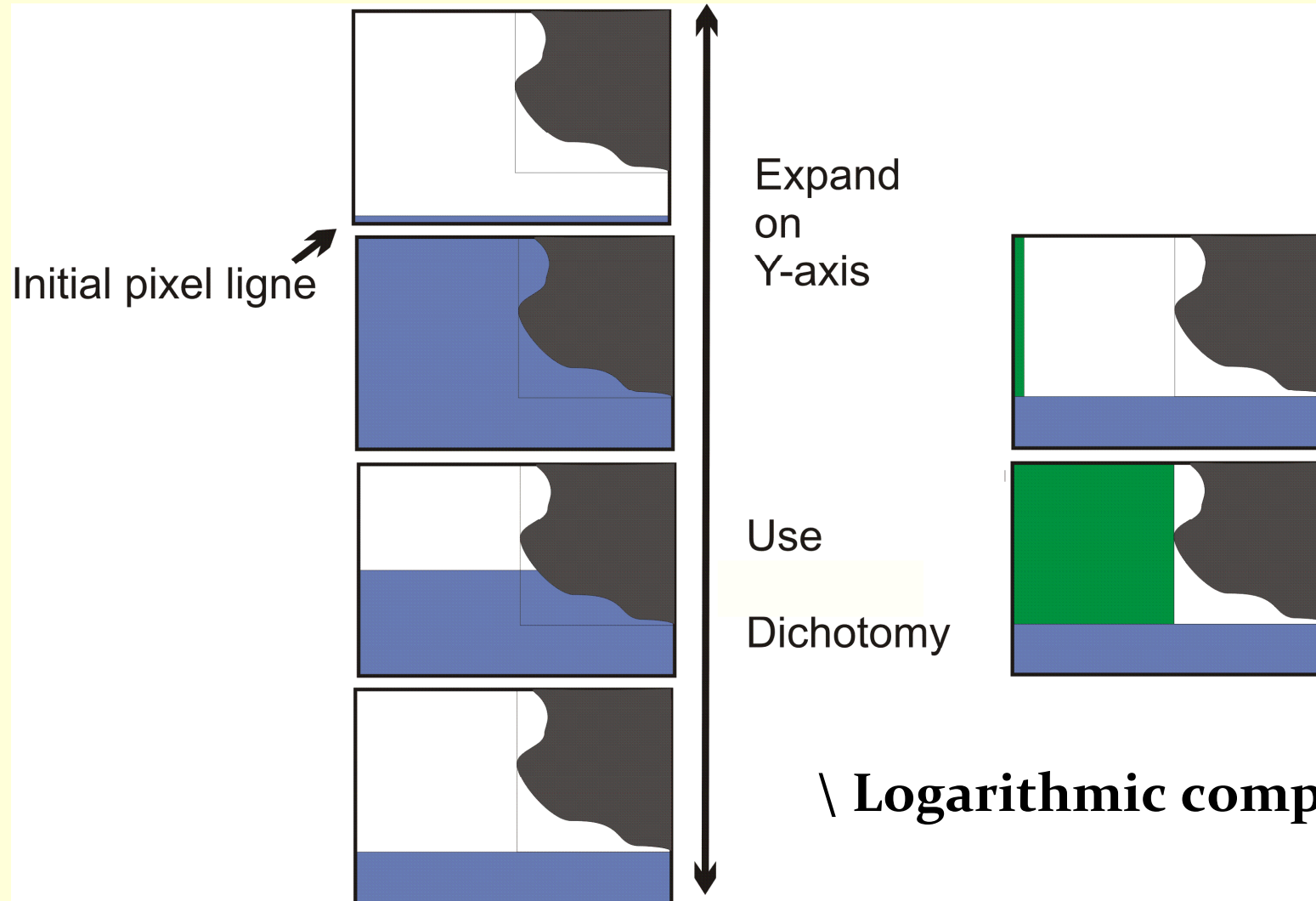




# Fourth method : Algorithm

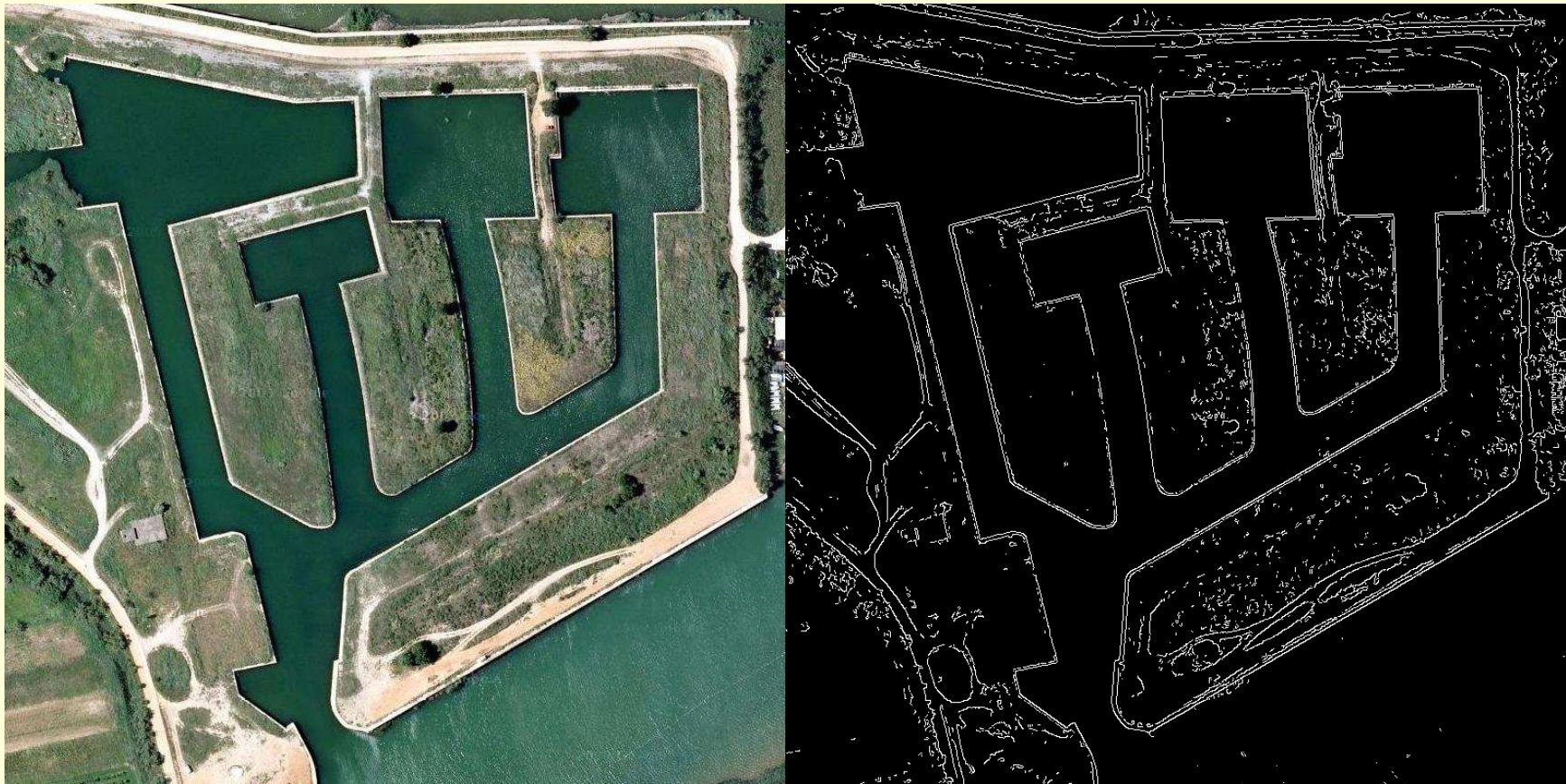


# Fourth method : Algorithm



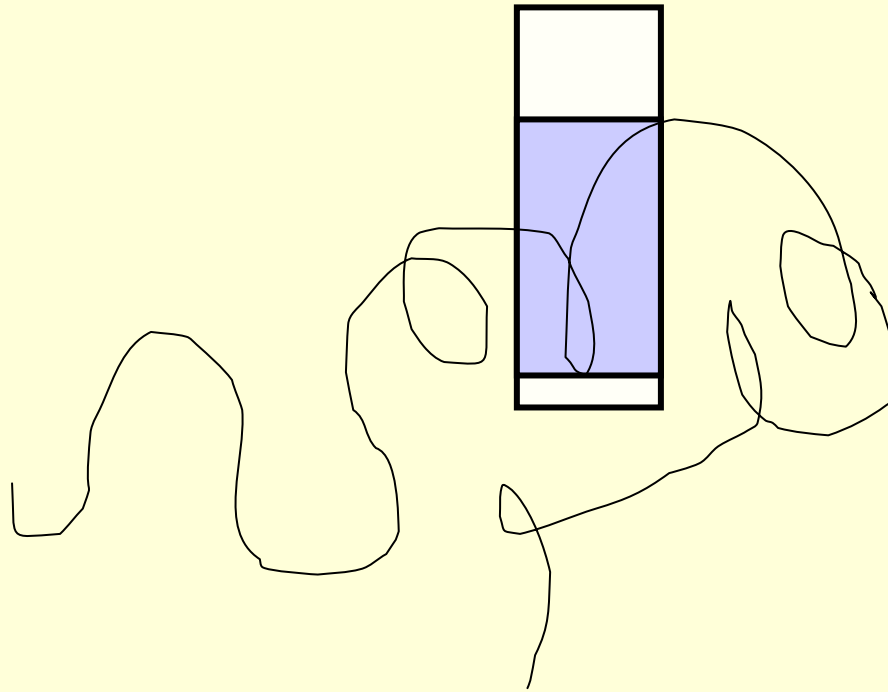
# Applications

## Localization (OpenCV Canny)



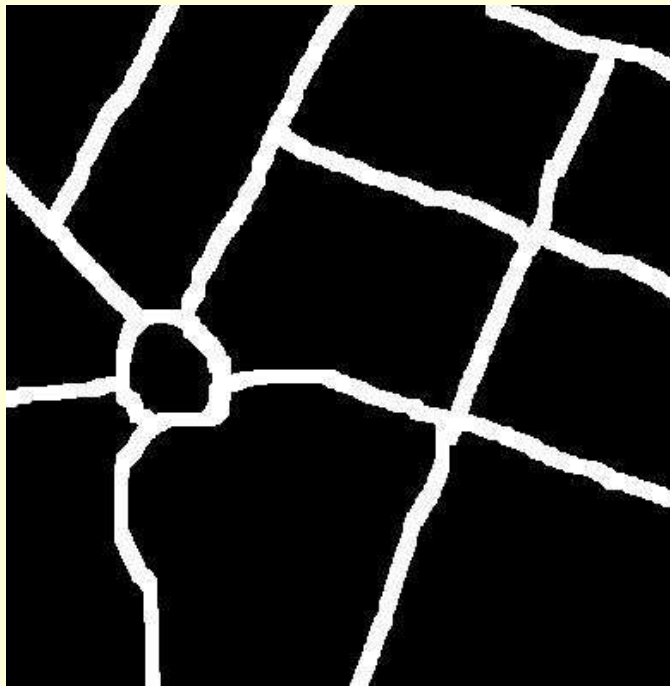
# Applications

Normal contractions



# Applications

Navigating in the city with **compass** and **speedometer only** (No GPS)



# Conclusion

- Image Contractor
- Promising for parallel computing hardware implementation (use Integers)
- Logarithmic complexity (on PC)
- Some real applications in localization or simply any contractions.
- The main drawback is storage



# ?Questions?

