Using interval methods in the context of robust localization of underwater robots

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Abstract— **In this paper we will apply interval methods to solve the problem of robust localization of an underwater robot. The localization problem is cast into a constraint satisfaction problem (CSP) where constraint propagation algorithms are particularly efficient. The method is designed to work in real environments with numerous outliers. Besides, we used a new approach to represent the map by a binary image. This allow us to represent even unstructured maps. We tested the algorithm on a real data set gathered by an underwater robot in a marina located in Costa Brava.**

CONTENTS

I. INTRODUCTION

There are many ways to address a problem of robot localization. Most of the proposed solutions are based on probabilistic estimation techniques (Kalman filtering, Bayesian estimation, particle filters, see [Thrun *et al.*, 2005]) which aim at blending data with some state equations of the robot. For the experiment to be presented, we will use a set-membership approach. In this formulation, both input data and computed position are represented by their respective membership sets. Constraints between the position of the robot and the sensor observations are used to contract the actual position set *i.e.* reduce its size thus increasing the estimates precision.

Set-membership methods have often been considered for the localization of robots (see, *e.g.*, [Meizel *et al.*, 1996], [Halbwachs and Meizel, 1996], in the case where the problem is linear and also [Caiti *et al.*, 2002] when the robot is underwater). In situations where strong nonlinearities are involved, interval analysis has been shown to be useful (see, *e.g.*, [Meizel *et al.*, 2002], where the first localization of an actual robot has been solved with interval methods). Another strong point of set membership methods is the ability to deal with outliers (see [Jaulin, 2009]). There are other robotics applications such as state estimation (see [Gning and Bonnifait, 2006]), dynamic localization of robots (see [Gning and Bonnifait, 2006]), SLAM (see [Le Bars *et al.*, 2010]), control of robots (see [Lydoire and Poignet, 2003] and [Vinas *et al.*, 2006]) or topology analysis of configuration spaces (see [Delanoue *et al.*, 2006]) where interval constraint propagation methods have been successful.

In this article we will present an example of robust localization using set membership methods. We will also propose an original method to use maps in the form of an binary image by constructing a contractor (see [Chabert and Jaulin, 2009] and [Jaulin *et al.*, 2001]) on this image. We tested the algorithms on a data set derived from experiments carried out in a marina located in the Costa Brava (Spain) by Girona university AUV.

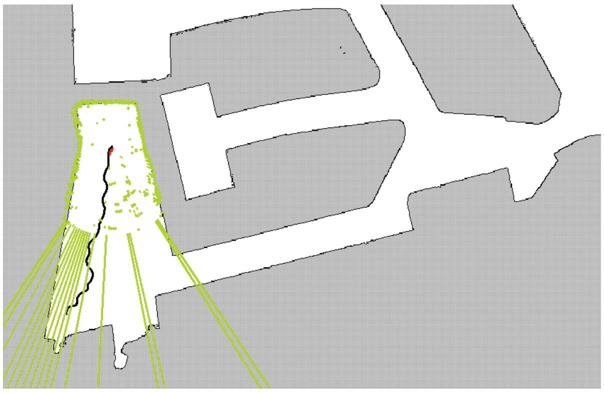

Fig. 1. Sonar measures sample

II. LOCALIZATION EXPERIMENT

The localization experiment to be considered here has been designed in order to illustrate a method for underwater SLAM (see [Ribas *et al.*, 2008]). The data was gathered during an extensive survey of a abandoned marina in the Costa Brava (Spain). Girona university *Ictineu* AUV gathered a data set along a $600m$ trajectory which included a small loop around the principal water tank and a $200m$ straight path through an outgoing canal. The data set included measurements from the Imaging Sonar (a Tritech Miniking), DVL - Doppler Velocity Log - (a SonTek Argonaut) and MRU - Motion Reference Unit - sensors (Xsens MTi). For validation purposes, the vehicle was operated close to the surface attached to a GPS equipped buoy used for registering the real trajectory (ground truth). Figure 1 shows a sample of sonar data, the green dots show where the sonar beam found an obstacle which is actually a wall from the marina. As one can see, the data is noisy and contains a lot of outliers. The long segments mean that obstacles are out of range i.e. beyond $50m$.

III. THE LOCALIZATION PROBLEM IS A CSP

A. Setting the problem into equations

Consider a system characterized by discrete-time dynamic equations.

$$\begin{array}{l} \mathbf{f_k} : \mathbb{R}^m \to \mathbb{R}^m, \mathbf{g_k} : \mathbb{R}^m \to \mathbb{R}^\ell \\ \mathbf{x_{k+1}} = \mathbf{f_k}(\mathbf{x_k}) \\ \mathbf{y_k} = \mathbf{g_k}(\mathbf{x_k}). \end{array} \tag{1}$$

In our case, $\mathbf{x}_k$ is the robots pose, $\mathbf{f_k}$ characterizes robots dynamics and $\mathbf{y}_k$ is the output vector. $\mathbf{y}_k$ and $\mathbf{x}_k$ are related by the observation function $\mathbf{g_k}$ which express in our case geometrical relations between the position, the measures and the map. Denote by $\mathbb{X}_k, \mathbb{Y}_k$ the sets containing $\mathbf{x_k}, \mathbf{y_k}$ respectively.

Using the recursive equation (1), we can easily prove that the system satisfies

$$\left\{ \begin{array}{l} \mathbf{x_{k+1}} = \mathbf{f_k} \circ \mathbf{g_k^{-1}}(\mathbf{y_k}) \\ \mathbf{x_{k+1}} = \mathbf{f_k} \circ \mathbf{f_{k-1}} \circ \mathbf{g_{k-1}^{-1}}(\mathbf{y_{k-1}}) \\ \ldots \\ \mathbf{x_{k+1}} = \mathbf{f_k} \circ \mathbf{f_{k-1}} \circ \ldots \mathbf{f_{k-n}} \circ \mathbf{g_{k-n}^{-1}}(\mathbf{y_{k-n}}) \\ \mathbf{x_{k+1}} = \mathbf{f_k}(\mathbf{x_k}) \\ \mathbf{x_{k+1}} \in \mathbb{R}^m, \mathbf{x_k} \in \mathbb{X}_k, \mathbf{y_i} \in \mathbb{Y}_i, i \in [k-n..k]. \end{array} \right. \tag{2}$$

Thus the problem of localization becomes a CSP i.e. Constraint Satisfaction Problem.

B. Robustness

More generally speaking, we can write the problem as follows

$$\left\{ \begin{array}{l} \mathbf{f}_0(\mathbf{x}) = \mathbf{0} \\ \mathbf{f}_1(\mathbf{x}) = \mathbf{0} \\ \ldots \\ \mathbf{f}_n(\mathbf{x}) = \mathbf{0} \end{array} \right. \quad \begin{array}{l} \mathbf{f}_i : \mathbb{R}^m \to \mathbb{R}^{n_i}, \\ \mathbf{x} \in \mathbb{X}. \end{array} \tag{3}$$

The solution of this CSP is

$$\mathbb{S} = \{\mathbf{x} \in \mathbb{R}^m, \forall i \in [0..n], \mathbf{f}_i(\mathbf{x}) = \mathbf{0}\} \tag{4}$$

In some cases, this CSP doesn't admit any solution. In the context of localization, that will be the case when some of the measurements are erroneous i.e. we have outliers. Dealing with outliers has already been considered by several authors, in a set membership context (see, *e.g.*, [Norton and Verez, 1993], [Lahanier *et al.*, 1987], [Pronzato and Walter, 1996], [Kreinovich *et al.*, 2003] ,[Jaulin, 2009]). In this case, we define a solution set $\mathbb{S}_q$ where $\mathbf{x} \in \mathbb{S}_q$ satisfies only a part of the set of equation. We will call this problem a relaxed CSP.

Definition 1: A q-relaxed resolution of the CSP (3) is searching for a solution set $\mathbb{S}_q$ where $\mathbf{x} \in \mathbb{S}_q$ satisfies at least $n-q$ among n equations *i.e.* searching for the following solution set

$$\begin{array}{l} \mathbb{S}_q = \{\mathbf{x} \in \mathbb{R}^m, \exists \mathbb{I} \subset \{1,..,n\}, \\ \qquad card(\mathbb{I}) = n-q, \forall i \in \mathbb{I}, \mathbf{f}_i(\mathbf{x}) = 0\}. \end{array} \tag{5}$$

C. Using set membership methods

Set membership methods allow us to manipulate sets (see [Moore, 1979] and [Jaulin *et al.*, 2001]). As an example, those methods allow to compute intersection $\mathbb{A} = \mathbb{B} \cap \mathbb{C}$, union $\mathbb{A} = \mathbb{B} \cup \mathbb{C}$, set inversion $\mathbb{A} = f^{-1}(\mathbb{B})$, image of a set by a function $\mathbb{A} = f(\mathbb{B})$...

Denote by

$$\mathbb{X}_i = \mathbf{f}_i^{-1}(0). \tag{6}$$

We can deduce that solution of the CSP is

$$\mathbb{S} = \bigcap_{i \in \{0..n\}} \mathbb{X}_i$$

Following the definition of $\mathbb{S}_q$, the q-relaxed resolution of the CSP gives

$$\mathbb{S}_q = \bigwedge_{i \in \{0..n\}}^{\{q\}} \mathbb{X}_i = \{\mathbf{x} \in \mathbb{R}^m, \exists \mathbb{I} \subset \{1,..,n\}, \; card(\mathbb{I}) = n-q, \forall i \in \mathbb{I}, \mathbf{x} \in \mathbb{X}_i\} \tag{7}$$

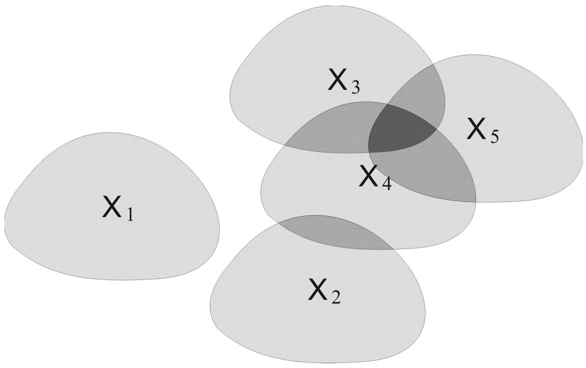

Fig. 2. Example of relaxed intersection

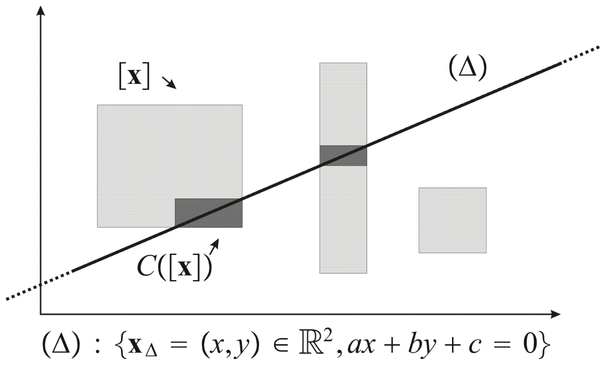

Fig. 3. Example of a constraint and the corresponding contractor

$\bigwedge\limits_{i\in\{0..n\}}^{\{q\}}$ is the q-relaxed intersection (see [Jaulin, 2009]). Figure 2 illustrates the q-relaxed intersection of sets $\mathbb{X}_1, \mathbb{X}_2, \mathbb{X}_3, \mathbb{X}_4, \mathbb{X}_5$. We have

$$\begin{aligned} \bigwedge_{i\in\{0..5\}}^{\{0\}} \mathbb{X}_i &= \bigcap_{i\in\{1..5\}} \mathbb{X}_i = \emptyset \\ \bigwedge_{i\in\{0..5\}}^{\{2\}} \mathbb{X}_i &= \mathbb{X}_3 \cap \mathbb{X}_4 \cap \mathbb{X}_5 \text{ (dark gray part in Fig. 2)} \\ \bigwedge_{i\in\{0..5\}}^{\{5\}} \mathbb{X}_i &= \bigcup_{i\in\{1..5\}} \mathbb{X}_i \end{aligned} \tag{8}$$

IV. Solving

A. Interval analysis

Interval analysis is used to create algorithms corresponding to operations on sets such as union, intersection, set inversion, ...

Sets are represented by a set of intervals or boxes which are defined below.

Definition 2: An *interval* is a connected and closed subset of $\mathbb{R}$.

Example 3: $\emptyset, \{-1\}, [-1, 1], [-1, \infty], \mathbb{R}$ are intervals.

Notation 4: If x is a real variable we denote by $[x]$ the interval containing this variable.

An interval has an upper and lower bound which we will note as follows$[x] = [x^-, x^+]$

$\mathbb{IR}$ is the set of all the intervals. Denote by $w([x]) = x^+ - x^-$ the width of the interval $[x]$.

Definition 5: A *box* of $\mathbb{R}^n$ is defined by a Cartesian product of intervals. A box can be also considered as an interval vector.

Example 6: $[1, 3] \times [2, 4]$ is a box of $\mathbb{R}^2$.

Notation 7: if $\mathbf{x}$ is a real variable vector we denote by $[\mathbf{x}]$ the box containing this variable. Denote by $[\mathbb{A}]$ the box enclosing the set $\mathbb{A}$.

Interval analysis resembles a lot real analysis except that we use intervals instead of real numbers. We can thus define binary operations on intervals $\{+, -, *, /, \max, \min\}$, interval functions....(see [Moore, 1979] and [Kearfott and Kreinovich, 1996] and [Jaulin *et al.*, 2001] for more details about interval analysis)

B. Resolution algorithm

Once we found the expression of our relaxed CSP, we can use the RSIVIA solver (see [Jaulin, 2009]) to compute the solution set. RSIVIA uses interval analysis and particularly the concept of contractors (see [Chabert and Jaulin, 2009] and [Jaulin *et al.*, 2001]). A contractor is an operator which applied on a box shrinks it in a specific manner. Contractors are used to represent and manipulate sets in algorithms. In fact, RSIVIA take each equation i.e. constraint from the CSP and transform it into a contractor.

Example : Consider a constraint $\mathbf{x} = (x, y) \in [\mathbf{x}], ax+by+c = 0$. Figure 3 shows how the constraint is transformed into a contractor C.

C. Image contractor

1) Introduction: As one can see in Figure 1, the map of the marina can be represented mostly by line segments and polygons. However, it is often cumbersome to extract those lines in order to use them as constraints in the CSP. As such, we propose the usage of an image contractor where the map is represented by a binary image. In the following part, we will show how to build an image contractor from a continuous binary image. The case where the image is discrete is easily derived from the continuous case.

Consider a continuous binary image defined by

$$f : \mathbb{R}^2 \to \{0, 1\} \tag{9}$$

Let $[\mathbf{x}]$ be a box of $\mathbb{R}^2$. The contractor C on the image is defined by

$$\begin{array}{l} \mathbb{E} = \{\mathbf{x} \in [\mathbf{x}], f(\mathbf{x}) = 1\} \\ C([\mathbf{x}]) = [\mathbb{E}] \end{array} \tag{10}$$

In fact, we choose the dark pixels $\mathbf{x}$ where $f(\mathbf{x}) = 1$ as the pixels of interest. Figure 4 shows the action of the contractor.

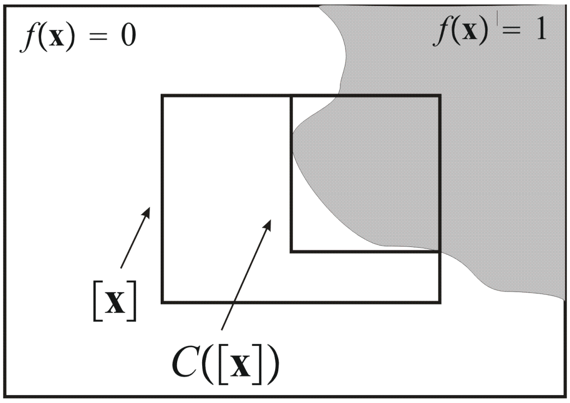

Fig. 4. Example of contraction on image

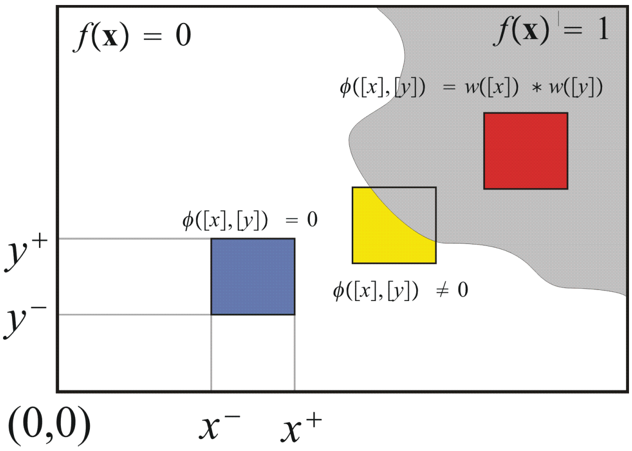

Fig. 5. Illustration of the role of ϕ

2) Implementation of the contractor: Before computing the contractor, we will characterize the intersection between the box $[\mathbf{x}]$ and the set defined by $f(\mathbf{x}) = 1$ i.e. the contractor set.

Consider the function

$$\begin{array}{l} \psi : \mathbb{R}^2 \to \mathbb{R} \\ \psi(a,b) = \int_0^a \int_0^b f(x,y)dxdy. \end{array} \tag{11}$$

ψ characterizes the quantity of pixels of interest in the box $[0,a] \times [0,b]$.

Generally speaking, the quantity of dark pixels in one box $[\mathbf{x}] = [x] \times [y]$ is defined by the function ϕ.

$$\begin{array}{l} \phi : \mathbb{IR}^2 \to \mathbb{R}, \\ \phi([x],[y]) = \int_{(x,y)\in[x]\times[y]} f(x,y)dxdy. \end{array} \tag{12}$$

In fact, as seen in Figure 5 for the blue box, ϕ can be obtained from ψ.

$$\begin{array}{l} \phi : \mathbb{IR}^2 \to \mathbb{R}, \\ \phi([x],[y]) = \psi(x^+,y^+) - \psi(x^-,y^+) \\ \qquad\qquad - \psi(x^+,y^-) + \psi(x^-,y^-). \end{array} \tag{13}$$

The idea is to compute ψ only once and obtain ϕ instantly for every box.

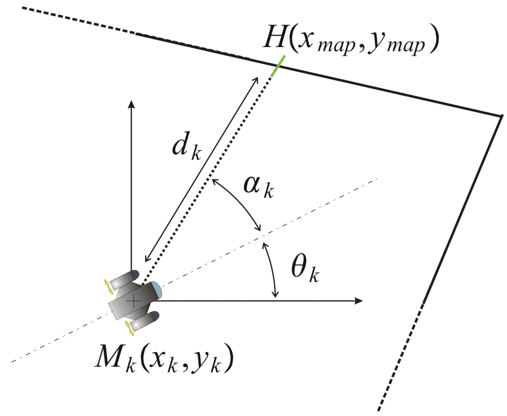

Fig. 6. Sonar data interpretation

In fact, ϕ characterizes the intersection between the box $[\mathbf{x}]$ and the set defined by $f(\mathbf{x}) = 1$ as shown in Figure 5. ϕ will be used to construct the contractor as explained in the following part.

3) Contraction algorithm: Denote by C the image contractor. Consider $[\mathbf{x}] = [x] \times [y] \in \mathbb{IR}^2$ and $C([\mathbf{x}]) = [\mathbf{x_c}]$. In our 2D case, we have

$$\begin{array}{l} x_c^- = \max(x \in [x], \phi([x^-,x],[y]) = 0) \\ x_c^+ = \min(x \in [x_c^-,x^+], \phi([x,x^+],[y]) = 0) \\ y_c^- = \max(y \in [y], \phi([x_c],[y^-,y]) = 0) \\ y_c^+ = \min(y \in [y_c^-,y^+], \phi([x_c],[y,y^+]) = 0). \end{array} \tag{14}$$

For the min and max calculus, we can use dichotomy thus the contractor complexity is logarithmic.

V. Experiment

A. Dynamic function

Robot's evolution is characterized by the following discrete-time dynamic equations.

$$\begin{array}{l} x_{k+1} = x_k + v_k \cos(\theta_k)dt \\ y_{k+1} = y_k + v_k \sin(\theta_k)dt. \end{array} \tag{15}$$

Where (x_k, y_k) is the position of the robot, v_k is its speed and θ_k is its orientation at time step k.

B. Observation function

Denote (x_{map}, y_{map}) the points of the map. With the sonar we measure the distance to first obstacle - here marina walls - along a vector defined by the sensor angle and robots heading (see Figure 6). There is a geometrical relationship (translation) between the position and the measure which is

$$(x_k, y_k) = (x_{map} - d_k \cos(\alpha_k + \theta_k), y_{map} - d_k \sin(\alpha_k + \theta_k)). \tag{16}$$

Where d_k is the distance to the wall and α_k is the sonar beam angle relative to the robot. (x_{map}, y_{map}) are the points of the map.

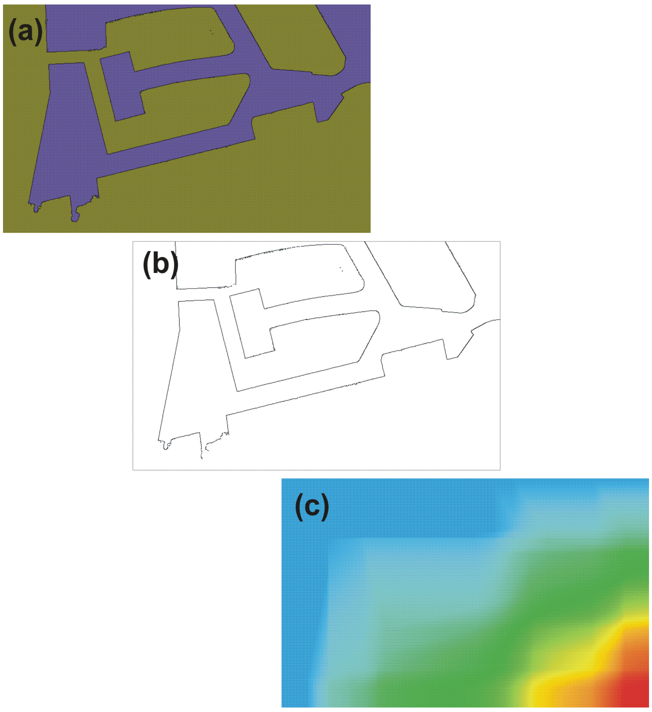

Fig. 7. Computing the image contractor

C. Map

As explained in part IV-C, the map will be represented as a binary image. The computation of the associated contractor on this image is showed on Figure 7 . First take a real map or image of the area (see Figure 7.a). Secondly, make edge detection or segmentation to obtain a binary image of the map (see Figure 7.b), Finally, compute ψ (see Figure 7.c) and the contractor is ready to use.

D. Results

Figure 8 shows a comparison between the reference GPS trajectory (in black) and the Dead Reckoning trajectory (in blue) which is obtained by merging DVL and MRU data. We can observe that Dead Reckoning trajectory suffers from an appreciable drift even causing it to go outside the canal. The trajectory computed using set membership approach is represented in Figure 9. The algorithm returns the trajectory as a set of boxes (in rose) but we usually take the center of the box as the actual position (in red). The red trajectory follows the GPS trajectory.

The algorithm execution was realtime on a core 2 duo.

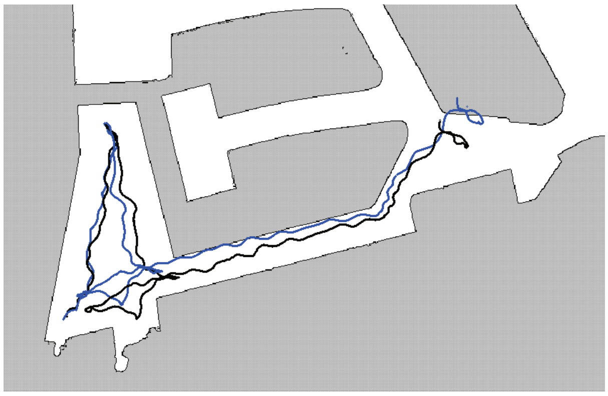

Fig. 8. Comparing GPS and Dead Reckoning trajectories

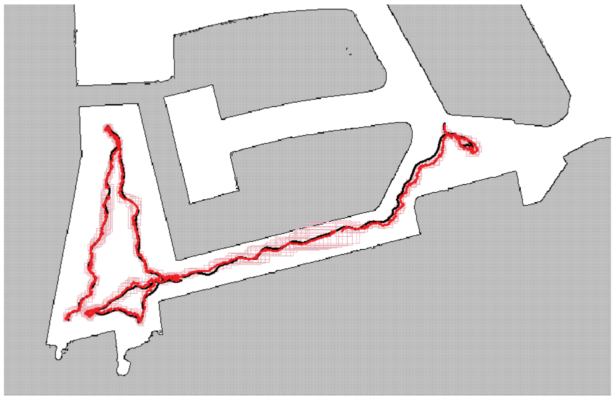

Fig. 9. Comparing GPS and set membership methods trajectories

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