Mise en équation d’un problème de SLAM robuste

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1ère année de thèse au sujet du SLAM par des méthodes ensemblistes dans le domaine de la robotique sous-marine
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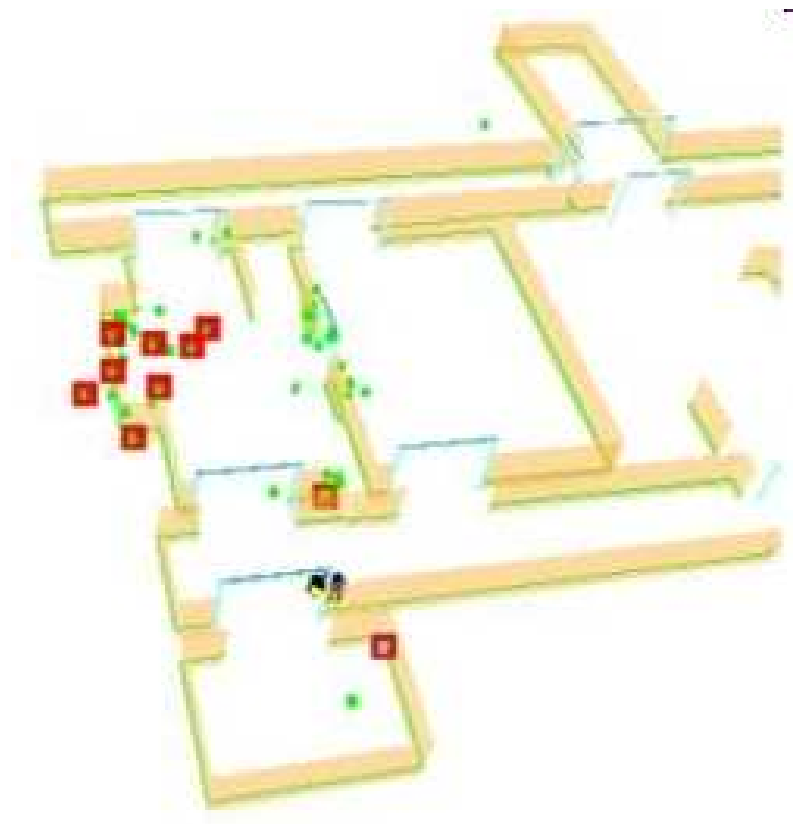
Problème de SLAM

- SLAM: localisation et cartographie simultanées
- Des équations **non-linéaires**
- Des **données aberrantes**
- Souvent des zones déterminées par des **polygones** et **segments**



Exemple d'un environnement d'évolution

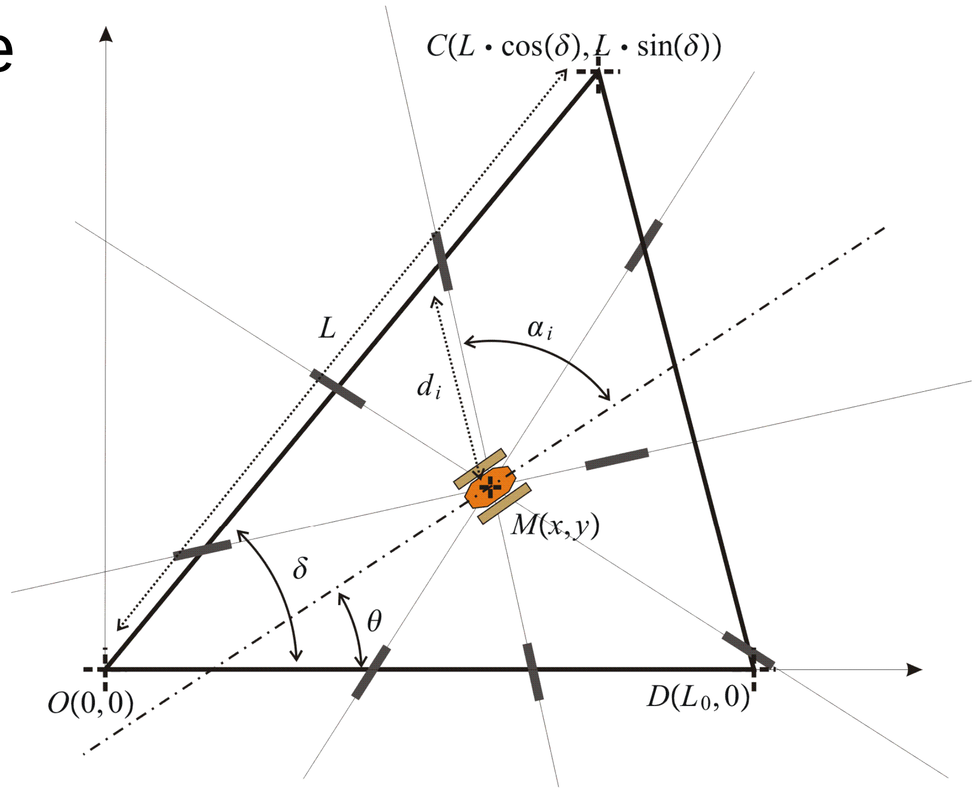
Problème simplifié

Les inconnues du problème

- pose x, y, θ
- carte L, δ

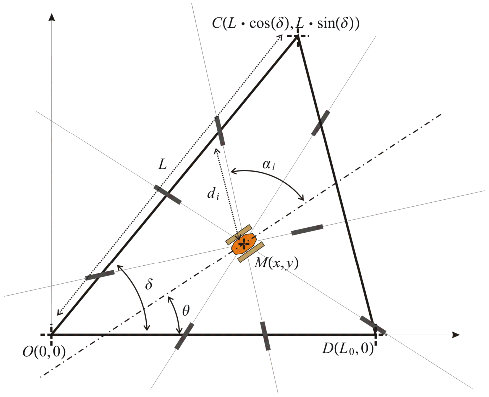
Les données

- n mesures télémétriques d_i



Mise en équation

- On souhaite mettre le problème sous la forme d'un CSP (Constraint Satisfaction Problem) ➞ Résolution avec des méthodes de propagation de contraintes (Méthodes ensemblistes)



➞

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➞

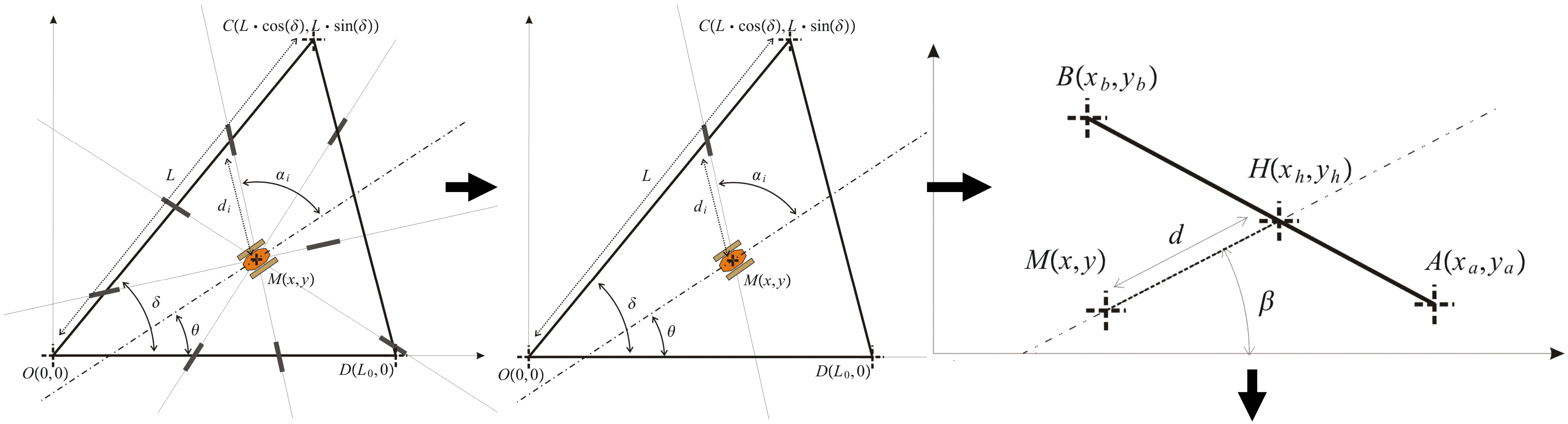
<u>Méthodes ensemblistes</u>

•Gèrent bien les non-linéarités

• permettent d'être **robuste** par rapport aux données aberrantes

Démo en fin de présentation

Mise en équation : détails des calculs



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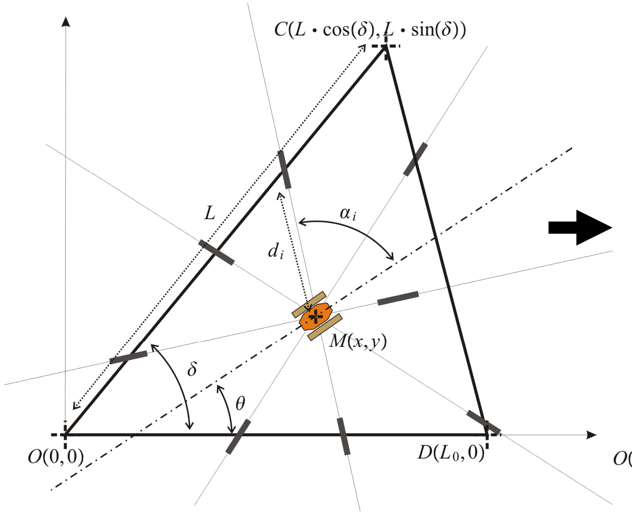

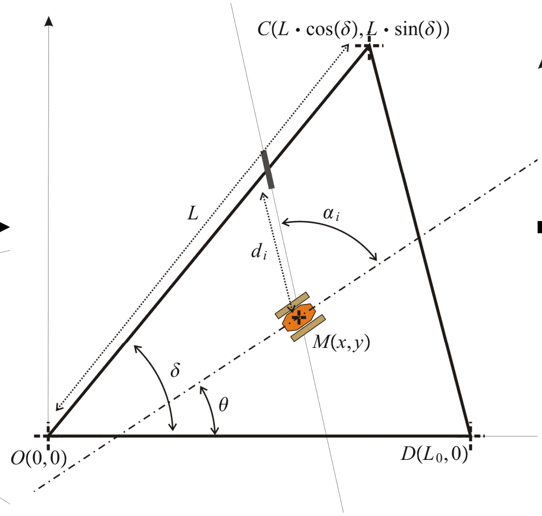

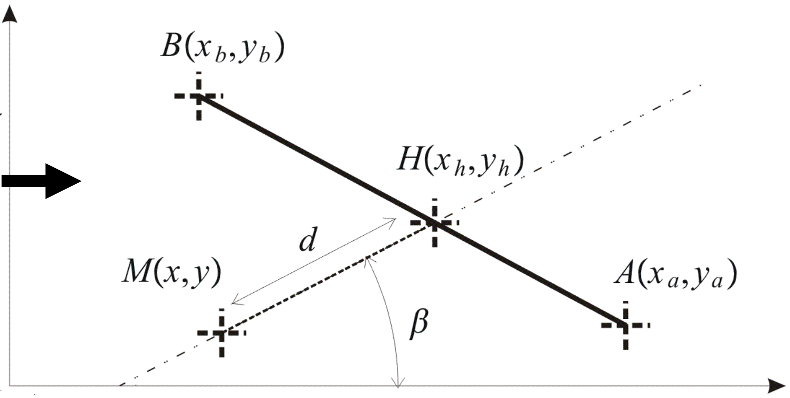

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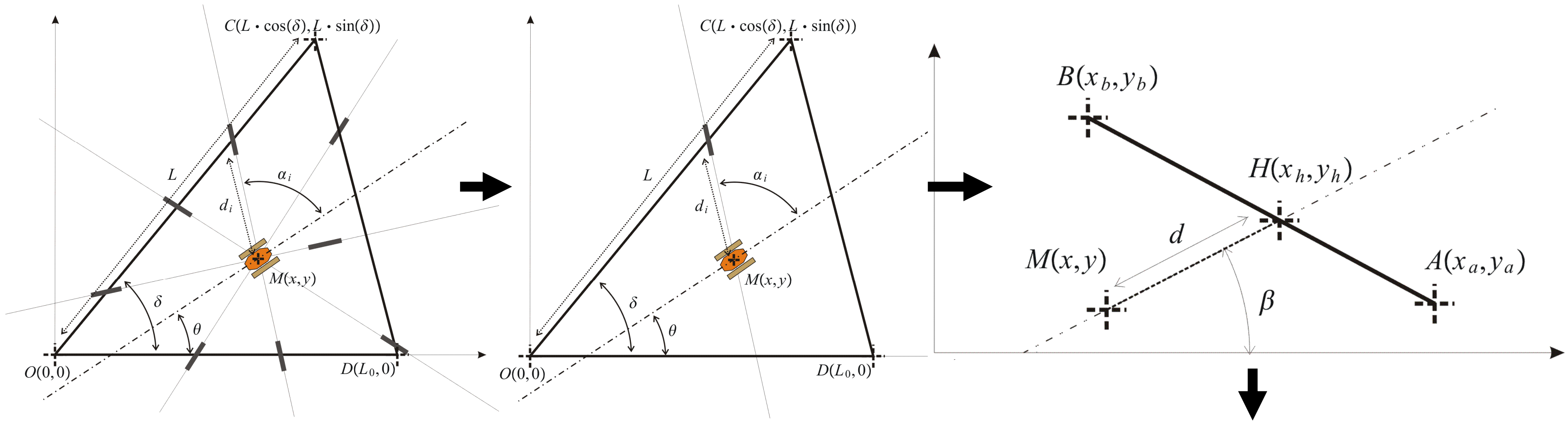

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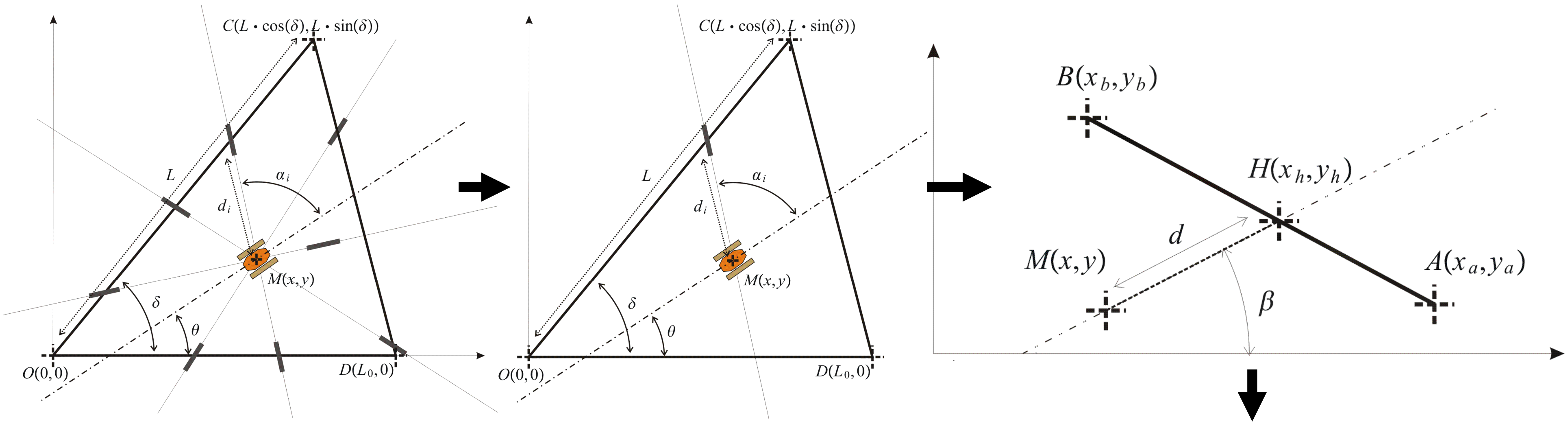

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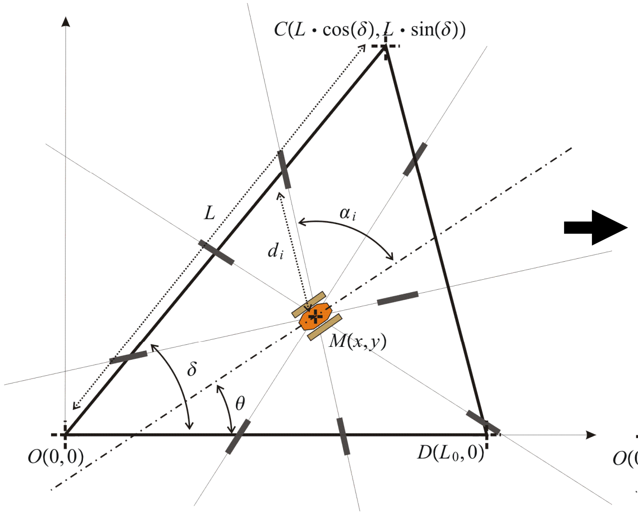

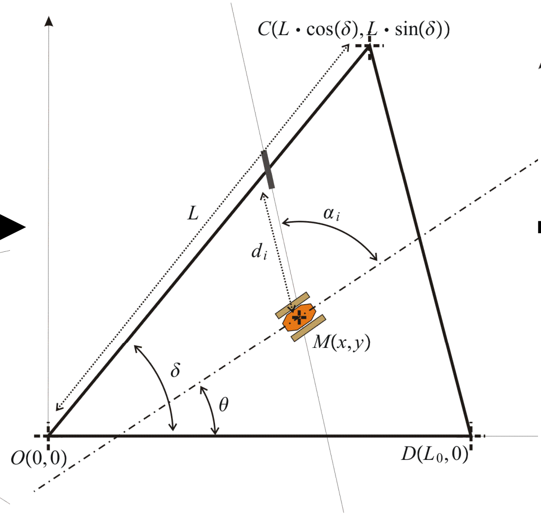

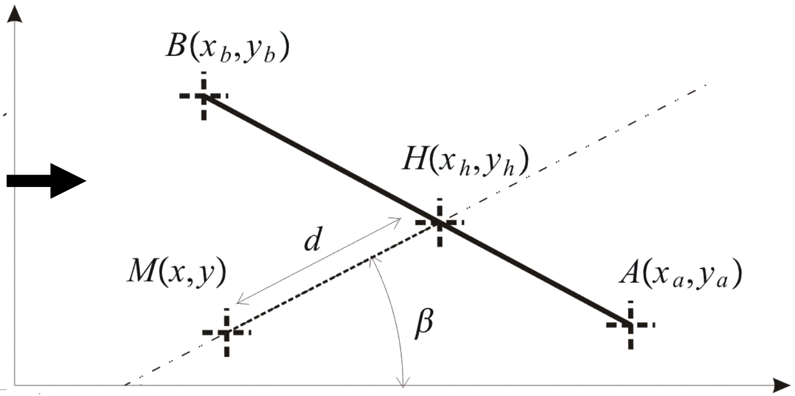

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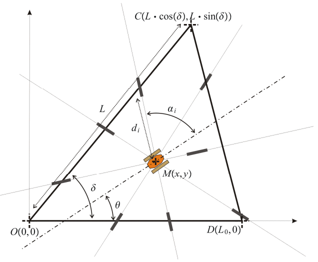

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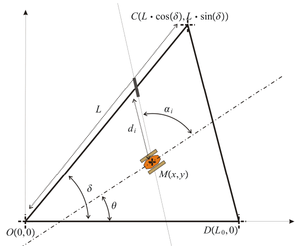

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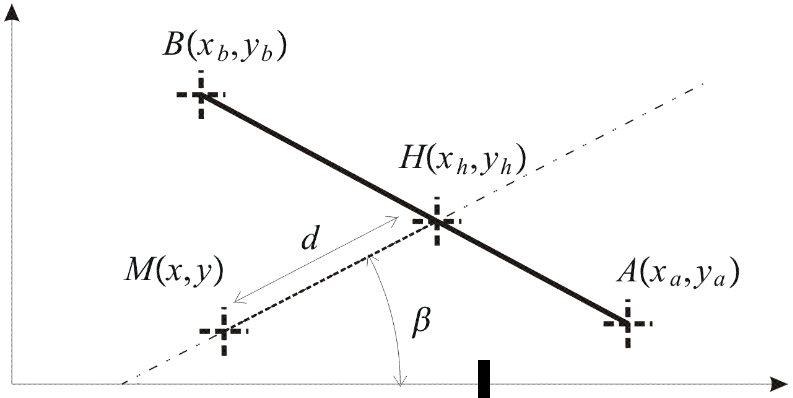

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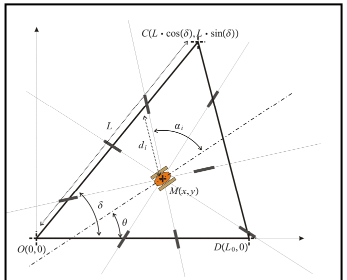

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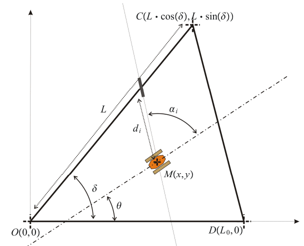

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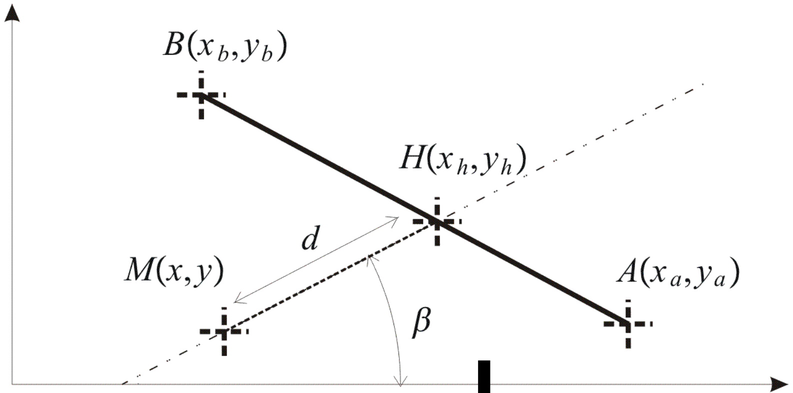

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$$\left\langle \overrightarrow{AH}, \overrightarrow{HB} \right\rangle > 0$$

$$\det\left(\overrightarrow{AB}, \overrightarrow{AM}\right) > 0$$

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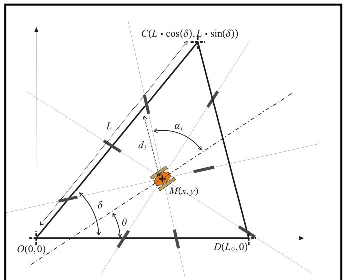

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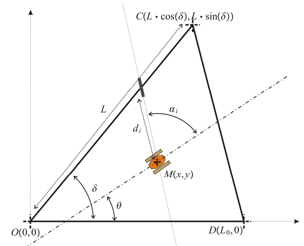

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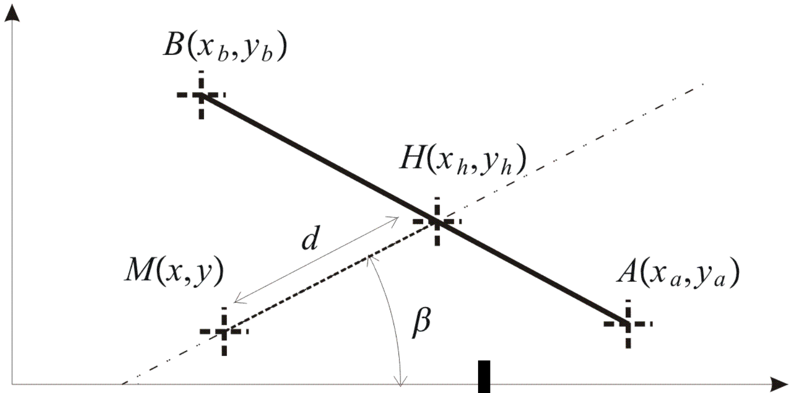

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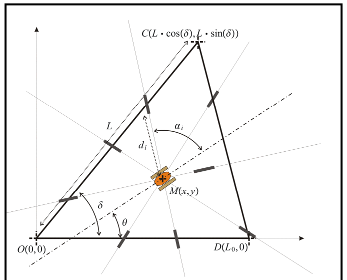

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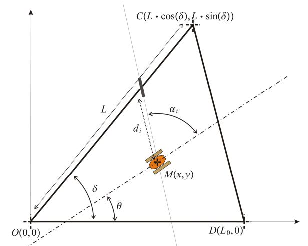

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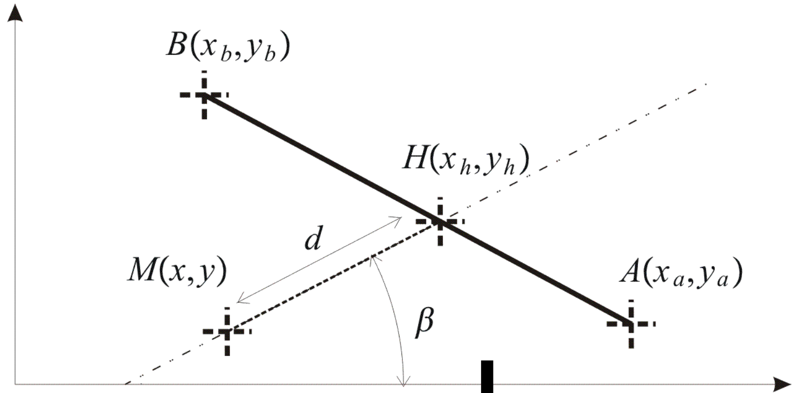

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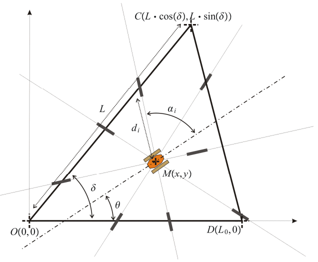

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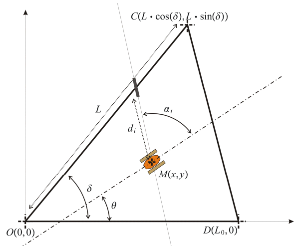

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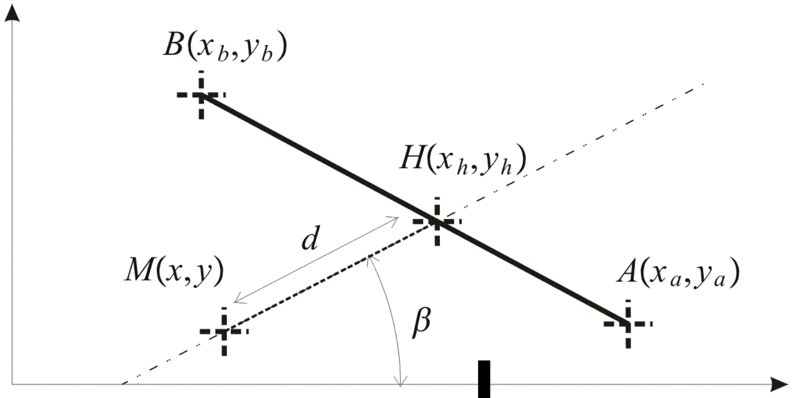

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Aparté sur le ET

- Le ET

Par exemple si on veut

$$X_1 = 0 \textbf{ ET } X_2 = 0 \textbf{ ET } X_3 = 0$$

Ceci est équivalent à dire que

$$|X_1| + |X_2| + |X_3| = 0$$

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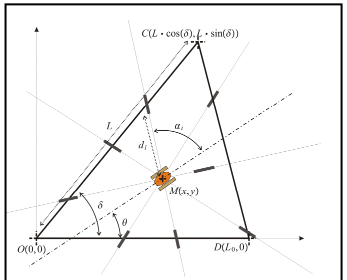

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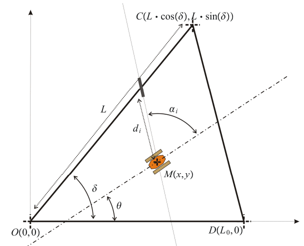

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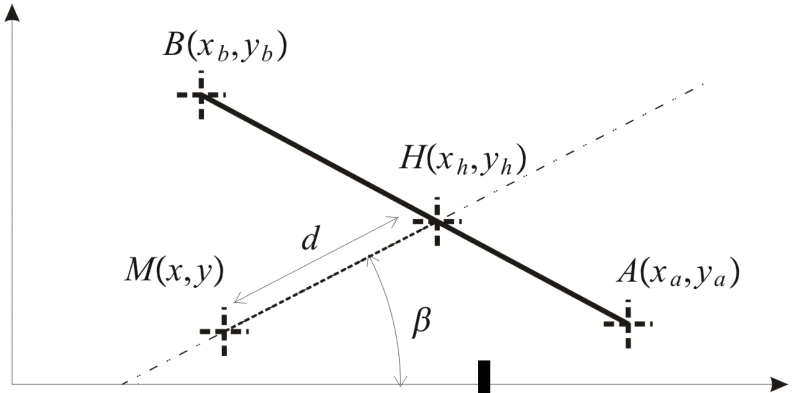

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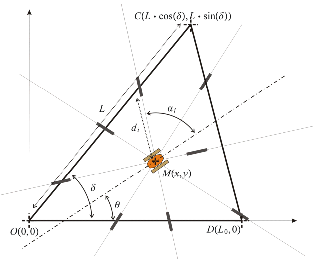

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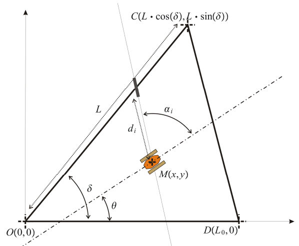
↕



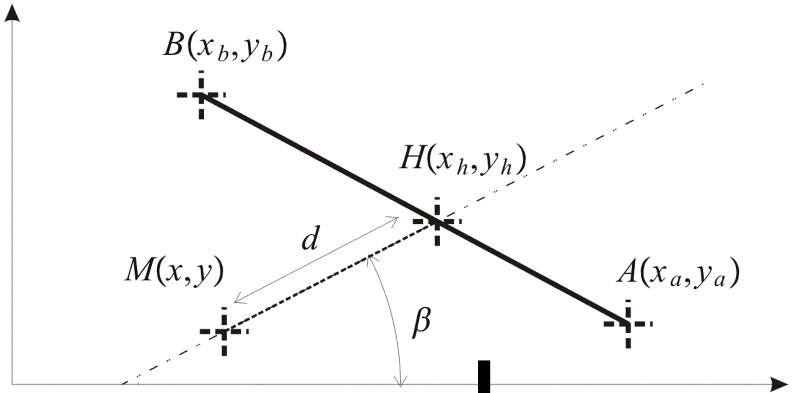
$$\begin{cases} \mathbf{f}_{ieme\_mesure}(\mathbf{x}) = 0, \\ \mathbf{x} = \begin{pmatrix} \mathbf{p} \\ \mathbf{c} \\ \mathbf{v} \end{pmatrix}, \\ \mathbf{f}_{ieme\_mesure} : \mathbb{R}^q \rightarrow \mathbb{R}^m, \\ \mathbf{x} \in [\mathbf{x}]. \end{cases}$$

↔

$$\begin{cases} \mathbf{f}_{segment\_ab}(\mathbf{x}) = 0, \\ \mathbf{x} = \begin{pmatrix} \mathbf{p} \\ \mathbf{c} \\ \mathbf{v} \end{pmatrix}, \\ \mathbf{f}_{segment\_ab} : \mathbb{R}^q \rightarrow \mathbb{R}^s, \\ \mathbf{x} \in [\mathbf{x}]. \end{cases}$$

↑

$$\begin{cases} f_1(x,y,x_a,y_a,x_b,y_b,d,\beta) - z_1 = 0 \\ f_2(x,y,x_a,y_a,x_b,y_b,d,\beta) - z_{p2} - z_2 = 0 \\ f_3(x,y,x_a,y_a,x_b,y_b,d,\beta) - z_{p3} - z_3 = 0 \\ |z_1| + |z_2| + |z_3| - z_{ab} = 0 \\ z_{ab} = 0? \\ z_{ab} \in [-\infty,+\infty] \\ z_1 \in [-\infty,+\infty], z_2 \in [-\infty,+\infty], z_3 \in [-\infty,+\infty] \\ z_{p2} \in [0,+\infty], z_{p3} \in [0,+\infty] \end{cases}$$



↓

$$\det\left(\overrightarrow{AB},\overrightarrow{AH}\right) = 0$$

ET $\left\langle \overrightarrow{AH},\overrightarrow{HB} \right\rangle > 0$

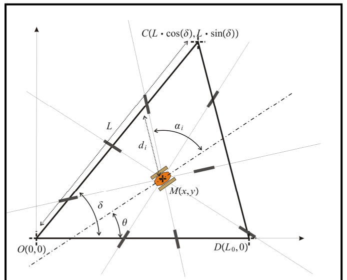
ET $\det\left(\overrightarrow{AB},\overrightarrow{AM}\right) > 0$

↓

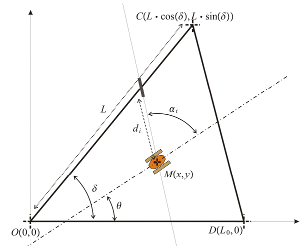
$$f_1(x,y,x_a,y_a,x_b,y_b,d,\beta) = 0$$
$$f_2(x,y,x_a,y_a,x_b,y_b,d,\beta) - z_{p2} = 0$$
$$f_3(x,y,x_a,y_a,x_b,y_b,d,\beta) - z_{p3} = 0$$
$$z_{p2} \in [0,+\infty], z_{p3} \in [0,+\infty]$$

←

Caractérisation du problème distance d'un point à un segment suivant une direction



$$\begin{cases} \mathbf{f}(\mathbf{x}) = \mathbf{0}, \\ \mathbf{x} = \begin{pmatrix} \mathbf{p} \\ \mathbf{c} \\ \mathbf{v} \end{pmatrix}, \\ \mathbf{f} : \mathbb{R}^q \to \mathbb{R}^p, \\ \mathbf{x} \in [\mathbf{x}]. \end{cases}$$



$$\begin{cases} \mathbf{f}_{ieme\_mesure}(\mathbf{x}) = 0, \\ \mathbf{x} = \begin{pmatrix} \mathbf{p} \\ \mathbf{c} \\ \mathbf{v} \end{pmatrix}, \\ \mathbf{f}_{ieme\_mesure} : \mathbb{R}^q \to \mathbb{R}^m, \\ \mathbf{x} \in [\mathbf{x}]. \end{cases}$$

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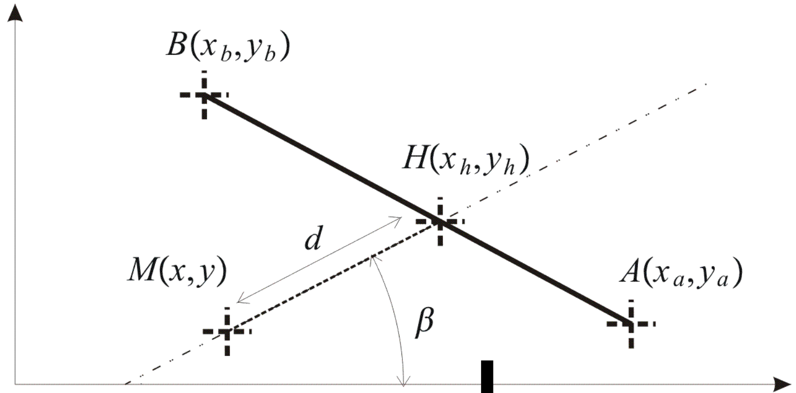
$$\begin{cases} \mathbf{f}_{seg\_ab}(\mathbf{x}_{seg}, z_{ab}) = \mathbf{0} \\ z_{ab} = 0? \\ z_{ab} \in [-\infty, +\infty], \\ \mathbf{x}_{seg} \in [\mathbf{x}_{seg}] \end{cases}$$



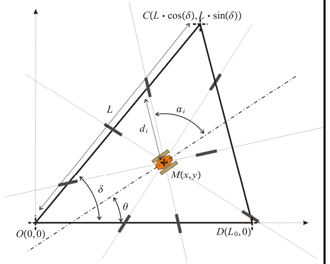
$$\det\left(\overrightarrow{AB}, \overrightarrow{AH}\right) = 0$$

ET $\left\langle \overrightarrow{AH}, \overrightarrow{HB} \right\rangle > 0$

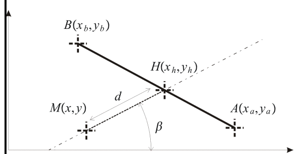
ET $\det\left(\overrightarrow{AB}, \overrightarrow{AM}\right) > 0$

$$f_1(x, y, x_a, y_a, x_b, y_b, d, \beta) = 0$$
$$f_2(x, y, x_a, y_a, x_b, y_b, d, \beta) - z_{p2} = 0$$
$$f_3(x, y, x_a, y_a, x_b, y_b, d, \beta) - z_{p3} = 0$$
$$z_{p2} \in [0, +\infty], z_{p3} \in [0, +\infty]$$

Mise en équation



$$\begin{cases} \mathbf{f}(\mathbf{x}) = \mathbf{0}, \\ \mathbf{x} = \begin{pmatrix} \mathbf{p} \\ \mathbf{c} \\ \mathbf{v} \end{pmatrix}, \\ \mathbf{f} : \mathbb{R}^q \rightarrow \mathbb{R}^p, \\ \mathbf{x} \in [\mathbf{x}]. \end{cases}$$



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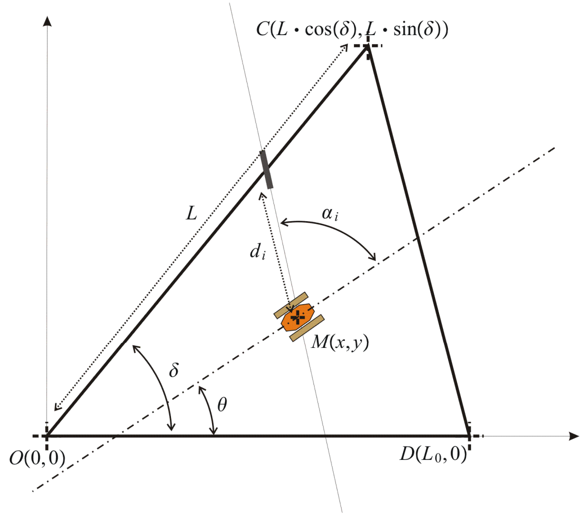


Pour chaque segment

$$\begin{cases} \mathbf{f}_{seg\_ab}(\mathbf{x}_{seg}, z_{ab}) = \mathbf{0} \\ z_{ab} = 0? \\ z_{ab} \in [-\infty, +\infty], \\ \mathbf{x}_{seg} \in [\mathbf{x}_{seg}] \end{cases}$$

Aparté sur le OU

- Le OU

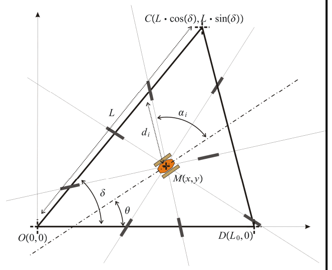
Par exemple si on veut

$$X_1 = 0 \textbf{ OU } X_2 = 0 \textbf{ OU } X_3 = 0$$

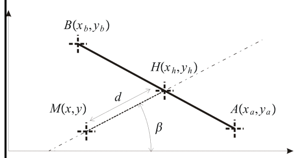
Ceci est équivalent à dire que

$$X_1 \bullet X_2 \bullet X_3 = 0$$

Mise en équation



$$\begin{cases} \mathbf{f}(\mathbf{x}) = \mathbf{0}, \\ \mathbf{x} = \begin{pmatrix} \mathbf{p} \\ \mathbf{c} \\ \mathbf{v} \end{pmatrix}, \\ \mathbf{f} : \mathbb{R}^q \to \mathbb{R}^p, \\ \mathbf{x} \in [\mathbf{x}]. \end{cases}$$



$$\begin{cases} \mathbf{f}_{segment\_ab}(\mathbf{x}) = 0, \\ \mathbf{x} = \begin{pmatrix} \mathbf{p} \\ \mathbf{c} \\ \mathbf{v} \end{pmatrix}, \\ \mathbf{f}_{segment\_ab} : \mathbb{R}^q \to \mathbb{R}^s, \\ \mathbf{x} \in [\mathbf{x}]. \end{cases}$$

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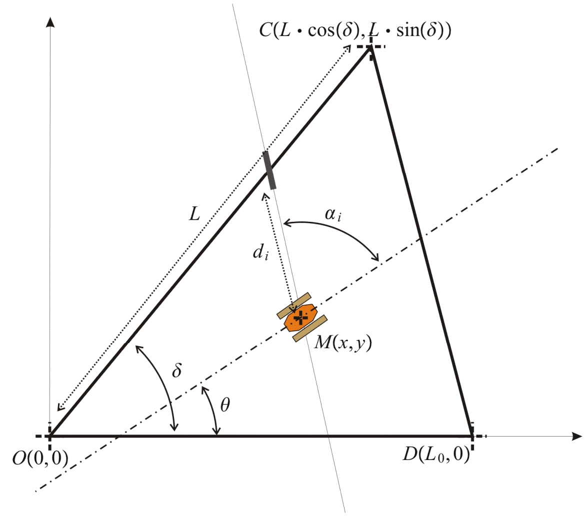
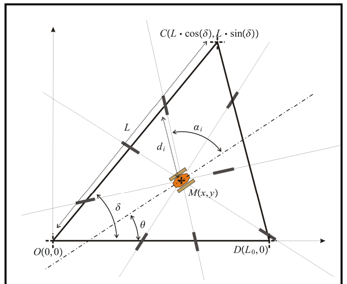


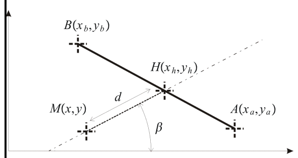
Pour chaque segment

$$\begin{cases} \mathbf{f}_{seg\_ab}(\mathbf{x}_{seg}, z_{ab}) = \mathbf{0} \\ z_{ab} = 0? \\ z_{ab} \in [-\infty, +\infty], \\ \mathbf{x}_{seg} \in [\mathbf{x}_{seg}] \end{cases}$$

Mise en équation



$$\begin{cases} \mathbf{f}(\mathbf{x}) = \mathbf{0}, \\ \mathbf{x} = \begin{pmatrix} \mathbf{p} \\ \mathbf{c} \\ \mathbf{v} \end{pmatrix}, \\ \mathbf{f} : \mathbb{R}^q \to \mathbb{R}^p, \\ \mathbf{x} \in [\mathbf{x}]. \end{cases}$$



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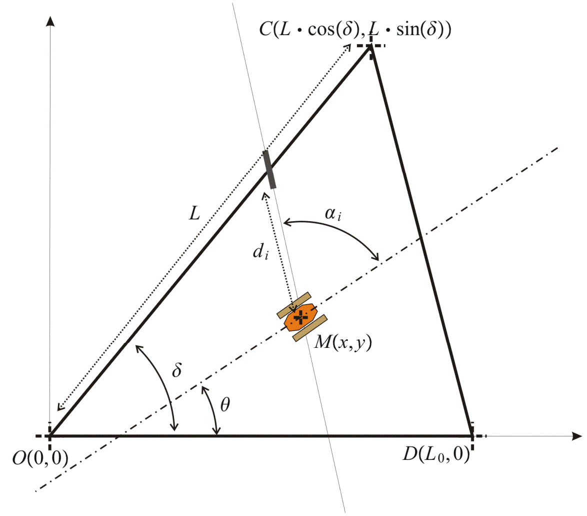
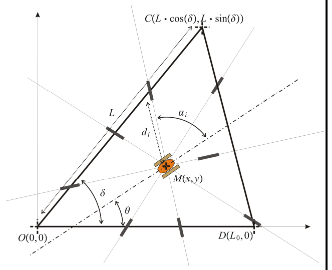


$$\begin{cases} \mathbf{f}_{iseg\_od}(\mathbf{x}, z_{iod}) = \mathbf{0} \\ \mathbf{f}_{iseg\_dc}(\mathbf{x}, z_{idc}) = \mathbf{0} \\ \mathbf{f}_{iseg\_co}(\mathbf{x}, z_{ico}) = \mathbf{0} \\ z_{iod} \cdot z_{idc} \cdot z_{ico} - z_i = 0 \\ z_i = 0? \\ z_i \in [-\infty, +\infty] \\ z_{iod} \in [-\infty, +\infty], z_{idc} \in [-\infty, +\infty], z_{ico} \in [-\infty, +\infty] \end{cases}$$

Pour chaque segment

$$\begin{cases} \mathbf{f}_{seg\_ab}(\mathbf{x}, z_{ab}) = \mathbf{0} \\ z_{ab} = 0? \\ z_{ab} \in [-\infty, +\infty], \\ \mathbf{x} \in [\mathbf{x}] \end{cases}$$

Mise en équation



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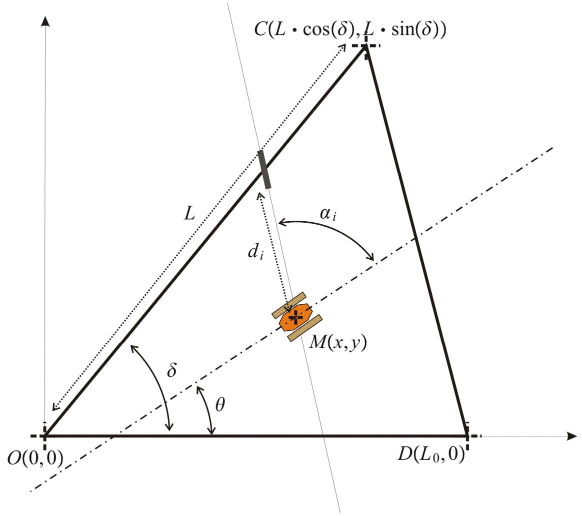

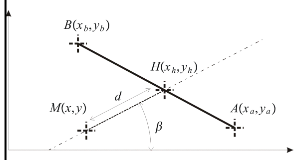

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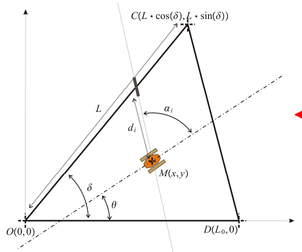
$$\begin{cases} \mathbf{f_i}(\mathbf{x}, z_i) = \mathbf{0} \\ z_i = 0? \\ z_i \in [-\infty, +\infty] \\ \mathbf{x} \in [\mathbf{x}] \end{cases}$$

Pour chaque segment

$$\begin{cases} \mathbf{f}_{seg\_ab}(\mathbf{x}, z_{ab}) = \mathbf{0} \\ z_{ab} = 0? \\ z_{ab} \in [-\infty, +\infty], \\ \mathbf{x} \in [\mathbf{x}] \end{cases}$$

Mise en équation



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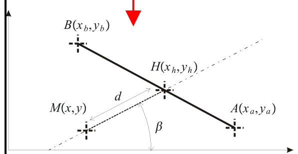

$$\begin{cases} \mathbf{f}_{segment\_ab}(\mathbf{x}) = 0, \\ \mathbf{x} = \begin{pmatrix} \mathbf{p} \\ \mathbf{c} \\ \mathbf{v} \end{pmatrix}, \\ \mathbf{f}_{segment\_ab} : \mathbb{R}^q \rightarrow \mathbb{R}^s, \\ \mathbf{x} \in [\mathbf{x}]. \end{cases}$$

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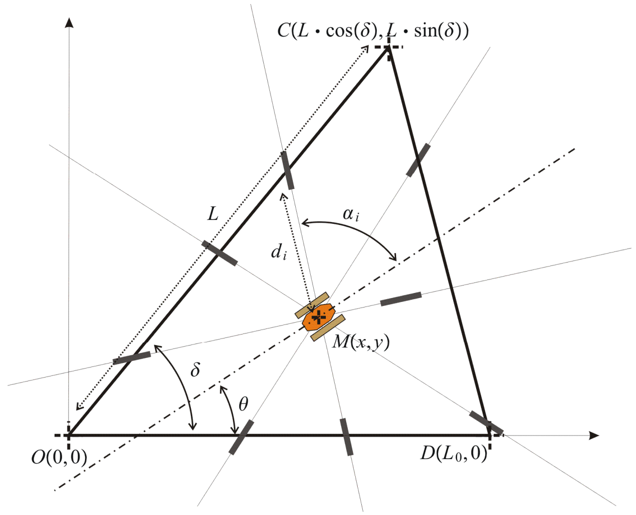
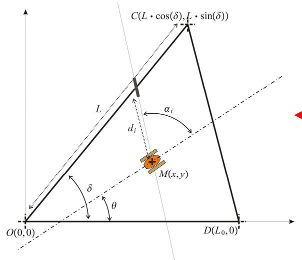


Sans données aberrantes

$$\begin{cases} \mathbf{f_i}(\mathbf{x}, z_i) = \mathbf{0} \\ z_i = 0? \\ z_i \in [-\infty, +\infty] \\ \mathbf{x} \in [\mathbf{x}] \end{cases}$$

Mise en équation



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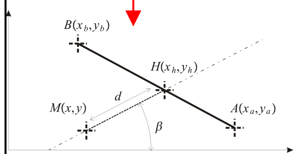

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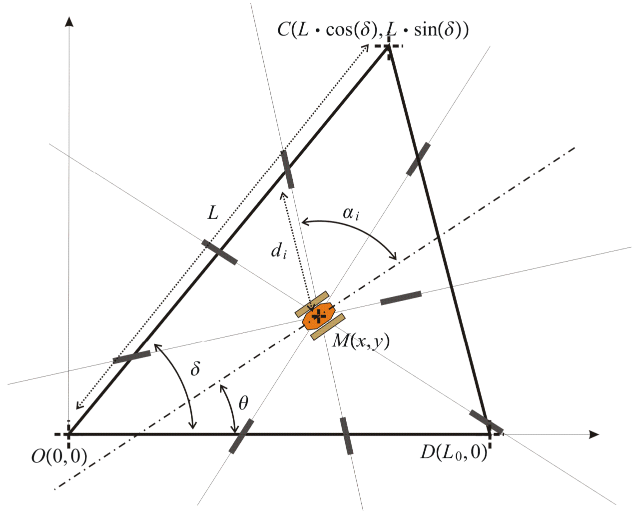


$$\begin{cases} \mathbf{f_1}(\mathbf{x}, z_i) = \mathbf{0} \\ \dots \\ \mathbf{f_i}(\mathbf{x}, z_i) = \mathbf{0} \\ \dots \\ \mathbf{f_n}(\mathbf{x}, z_n) = \mathbf{0} \\ |z_1| + |z_2| + \ldots + |z_n| = 0 \\ \forall i \in \{1..n\}, z_i \in [-\infty, +\infty] \\ \mathbf{x} \in [\mathbf{x}] \end{cases}$$

ET

Sans données aberrantes

$$\begin{cases} \mathbf{f_i}(\mathbf{x}, z_i) = \mathbf{0} \\ z_i = 0? \\ z_i \in [-\infty, +\infty] \\ \mathbf{x} \in [\mathbf{x}] \end{cases}$$

Aparté sur les polynômes symétriques

- Polynômes symétriques élémentaires

Définition $\phi_k(X_1,..X_n) = \sum_{1\leq i_0<...<i_k\leq n} X_{i_1}X_{i_2}\ldots X_{i_k}$

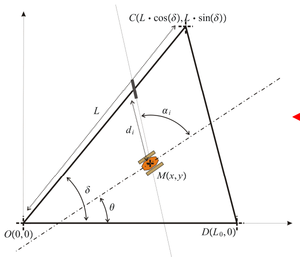
Exemple

$$\phi_0(X_1,..,X_4) = X_1 + X_2 + X_3 + X_4$$
$$\phi_1(X_1,..,X_4) = X_1X_2 + X_1X_3 + X_1X_4 + X_2X_3 + X_2X_4 + X_3X_4$$
$$\phi_2(X_1,..,X_4) = X_1X_2X_3 + X_1X_2X_4 + X_1X_3X_4 + X_2X_3X_4$$
$$\phi_3(X_1,..,X_4) = X_1X_2X_3X_4$$

Propriétés intéressantes si $\forall i \in \{1,..,n\}, X_i \in [0,+\infty]$

Si $\phi_\kappa(X_1,X_2,..,X_n) = 0$ *alors* $n-\kappa$ *variables parmi* $X_1,\ldots,X_n$ *sont égaux à zero*

Mise en équation



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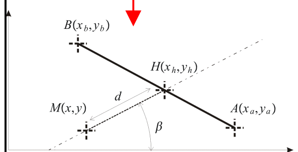

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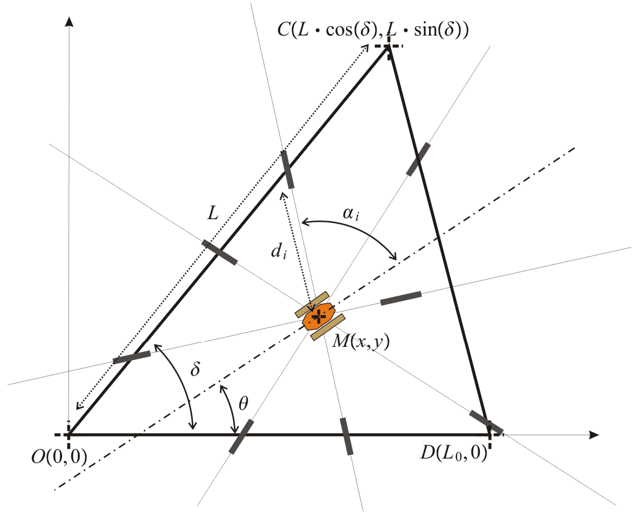
$$\begin{cases} \mathbf{f}(\mathbf{x}) = \mathbf{0}, \\ \mathbf{x} = \begin{pmatrix} \mathbf{p} \\ \mathbf{c} \\ \mathbf{v} \end{pmatrix}, \\ \mathbf{f} : \mathbb{R}^q \to \mathbb{R}^p, \\ \mathbf{x} \in [\mathbf{x}]. \end{cases}$$



$$\begin{cases} \mathbf{f_1}(\mathbf{x}, z_i) = \mathbf{0} \\ \dots \\ \mathbf{f_i}(\mathbf{x}, z_n) = \mathbf{0} \\ \dots \\ \mathbf{f_n}(\mathbf{x}, z_n) = \mathbf{0} \\ \phi_\kappa(|\mathbf{z}_1|, |\mathbf{z}_2|, \dots, |\mathbf{z}_n|) = \sum_{1 \le i_0 < \dots < i_\kappa \le n} |\mathbf{z}_{\mathbf{i_1}}| \cdot |\mathbf{z}_{\mathbf{i_2}}| \dots \cdot |\mathbf{z}_{\mathbf{i_\kappa}}| = 0 \\ \forall i \in \{1..n\}, z_i \in [-\infty, +\infty] \\ \mathbf{x} \in [\mathbf{x}] \end{cases}$$

k données aberrantes

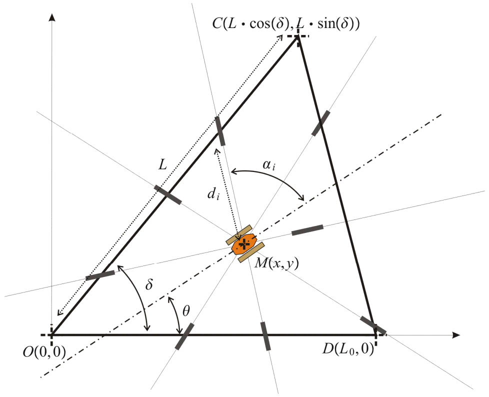
$$\begin{cases} \mathbf{f_i}(\mathbf{x}, z_i) = \mathbf{0} \\ z_i = 0? \\ z_i \in [-\infty, +\infty] \\ \mathbf{x} \in [\mathbf{x}] \end{cases}$$

CSP final

$$\left\{\begin{array}{l}
\ldots \\
\ldots \\
L_0 \cdot (y + d_i \cdot \sin(\theta + \alpha_i)) \ - z_{iod1} \ = \ 0 \\
(x + d_i \cdot \cos(\theta + \alpha_i))(L_0 - x - d_i \cdot \cos(\theta + \alpha_i)) \ - (y + d_i \cdot \sin(\theta + \alpha_i))^2 - z_{piod2} - z_{iod2} \ = \ 0 \\
L_0 \cdot y - z_{piod3} - z_{iod3} \ = \ 0 \\
|z_{iod1}| + |z_{iod2}| + |z_{iod3}| - z_{iod} \ = \ 0 \\
(L\sin(\delta) - L_0)(y + d_i \cdot \sin(\theta + \alpha_i)) \ + L\sin(\delta)(x + d_i \cdot \cos(\theta + \alpha_i) - L_0) - z_{idc1} \ = \ 0 \\
(x + d_i \cdot \cos(\theta + \alpha_i) - L_0)(L\cos(\delta) - x - d_i \cdot \cos(\theta + \alpha_i)) \ + (y + d_i \cdot \sin(\theta + \alpha_i))(L\sin(\delta) - y - d_i \cdot \sin(\theta + \alpha_i)) \ - z_{pidc2} - z_{idc2} \ = \ 0 \\
(L\cos(\delta) - L_0) \cdot y - L\sin(\delta)(x - L_0) - z_{pidc3} - z_{idc3} \ = \ 0 \\
|z_{idc1}| + |z_{idc2}| + |z_{idc3}| - z_{idc} \ = \ 0 \\
-L\cos(\delta)(y + d_i \cdot \sin(\theta + \alpha_i) - L\sin(\delta)) \ - L\sin(\delta)(x + d_i \cdot \cos(\theta + \alpha_i) - L\cos(\delta)) \ - z_{ico1} \ = \ 0 \\
(x + d_i \cdot \cos(\theta + \alpha_i) - L\cos(\delta))(-x - d_i \cdot \cos(\theta + \alpha_i)) \ + (y + d_i \cdot \sin(\theta + \alpha_i) - L\sin(\delta))(-y - d_i \cdot \sin(\theta + \alpha_i)) \ - z_{pico2} - z_{ico2} \ = \ 0 \\
-L\cos(\delta)(y - L\sin(\delta)) \ + L\sin(\delta)(x - L\cos(\delta)) \ - z_{pico3} - z_{ico3} \ = \ 0 \\
|z_{ico1}| + |z_{ico2}| + |z_{ico3}| - z_{ico} \ = \ 0 \\
z_{iod} \cdot z_{idc} \cdot z_{ico} - z_i \ = \ 0 \\
\ldots \\
\ldots \\
\phi_\kappa(|z_1|, |z_2|, \ldots, |z_n|) \ = \sum_{1 \le i_0 < \ldots < i_\kappa \le n} |z_{i_1}| \cdot |z_{i_2}| \ldots \cdot |z_{i_\kappa}| \ = \ 0 \\
x \in [x], y \in [y], \theta \in [\theta], L \in [L], \delta \in [\delta] \\
\forall i \in \{1..n\}, d_i \in [d_i], \{z_{i*}\ldots\} \ \subset [-\infty, +\infty], \{z_{pi*}\ldots\} \ \subset [0, +\infty]
\end{array}\right.$$

démo logicielle

- On souhaite mettre le problème sous la forme d'un CSP (Constraint Satisfaction Problem) → Résolution avec des méthodes de propagation de contraintes (Méthodes ensemblistes)



→

$$\begin{cases} \mathbf{f}(\mathbf{x}) = \mathbf{0}, \\ \mathbf{x} = \begin{pmatrix} \mathbf{p} \\ \mathbf{c} \\ \mathbf{v} \end{pmatrix}, \\ \mathbf{f} : \mathbb{R}^q \to \mathbb{R}^p, \\ \mathbf{x} \in [\mathbf{x}]. \end{cases}$$

→

Méthodes ensemblistes

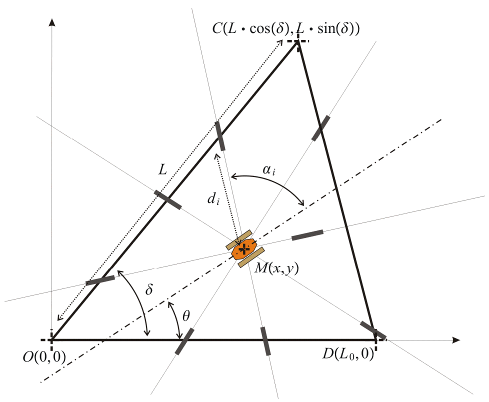
- Gèrent bien les non-linéarités
- permettent d'être **robuste** par rapport aux données aberrantes

Démo en fin de présentation

??? Questions ???

<u>résumé</u>

- On souhaite mettre le problème sous la forme d'un CSP (Constraint Satisfaction Problem) → Résolution avec des méthodes de propagation de contraintes (Méthodes ensemblistes)



→

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→

<u>Méthodes ensemblistes</u>

•Gèrent bien les non-linéarités

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Démo en fin de présentation